Simple harmonic motion

Learning outcomes

- define SHM and illustrate it with a variety of examples
- analyse SHM in terms of potential and kinetic energy
- describe effects of damping, forced vibrations and resonance
- interpret & use algebraic and graphical representations of SHM
- relate SHM to circular motion
- solve quantitative problems involving SHM

A brief history & contexts

The study of SHM started with Galileo's pendulum experiments (1638, *Two new sciences*).

Huygens, Newton & others developed further analyses.

- pendulum – clocks, seismometers
- ALL vibrations and waves - sea waves, earthquakes, tides, orbits of planets and moons, water level in a toilet on a windy day, acoustics, AC circuits, electromagnetic waves, vibrations molecular and structural e.g. aircraft fuselage, musical instruments, bridges.

Getting a feel for SHM 1

Careful observation of a big pendulum.

There is an equilibrium (rest) position. Displacement, *s*, is the distance from equilibrium.

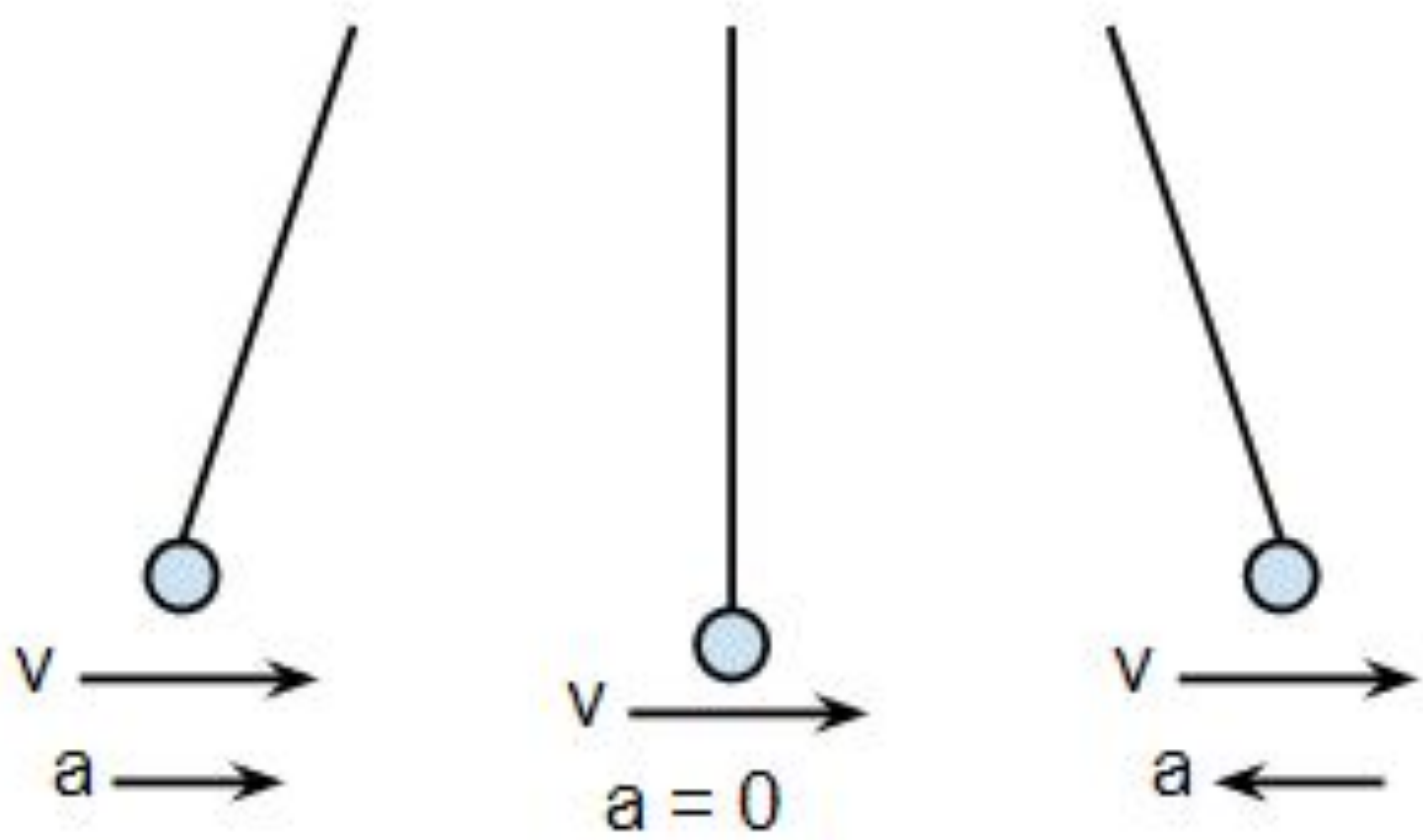
Discuss, in pairs

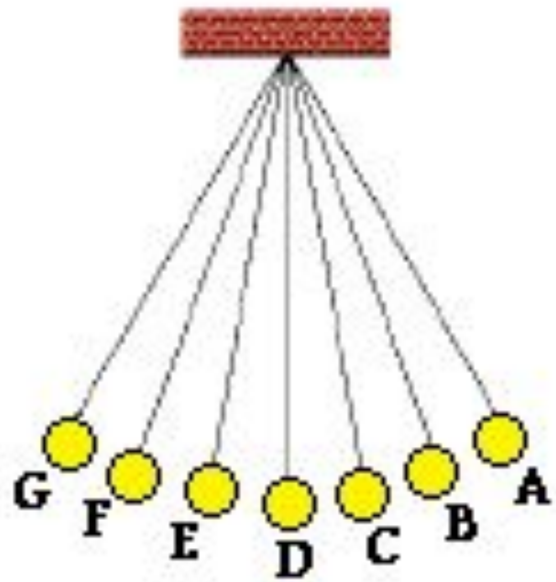
1. Where is the mass moving fastest, slowest?
2. Where is the mass's acceleration maximum, zero?
3. What causes the mass to overshoot its equilibrium position?
4. What forces act on the mass? When is the unbalanced force at a maximum value, zero?

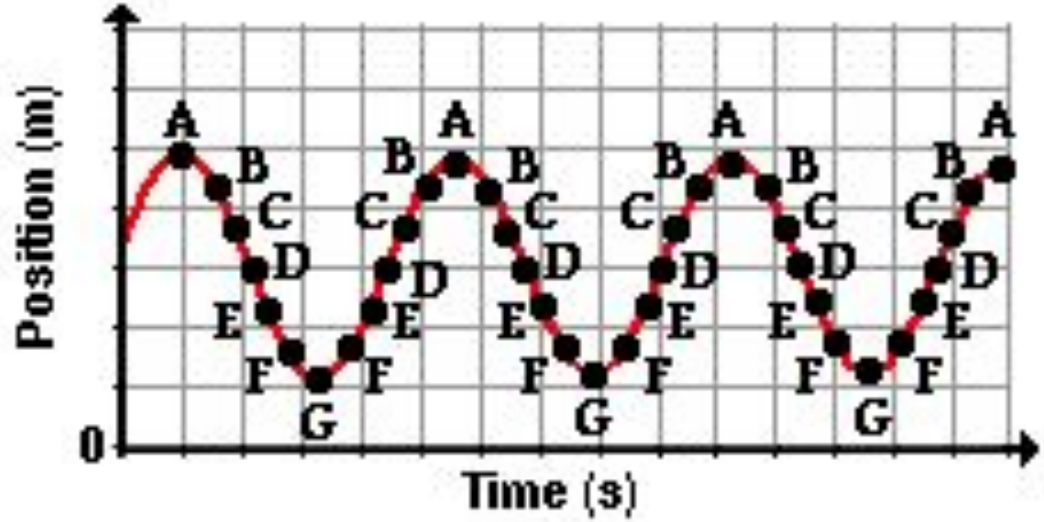
Getting a feel for SHM 2

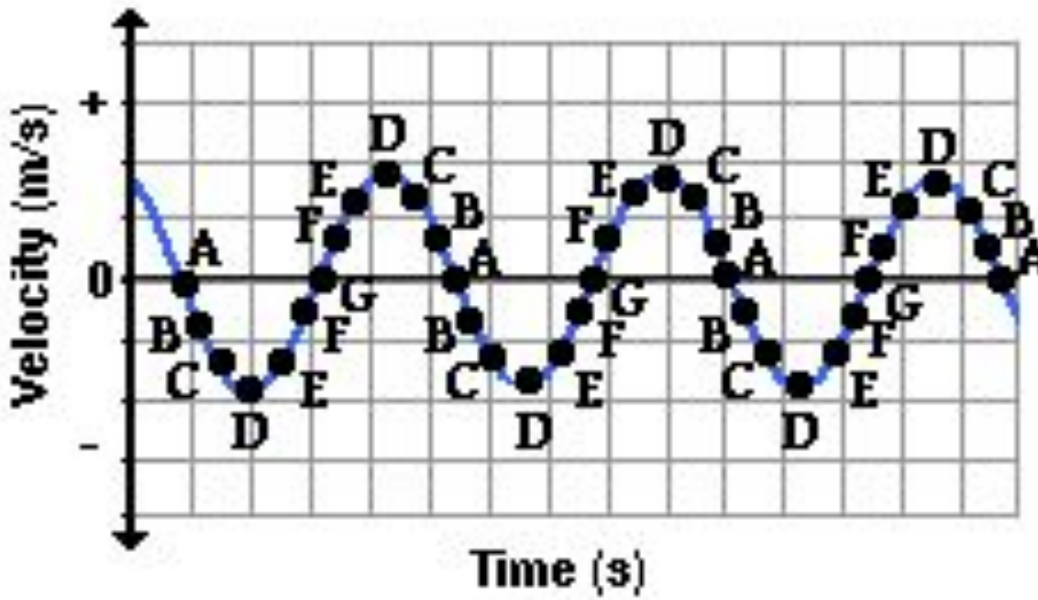
A circus of experiments (in small groups)

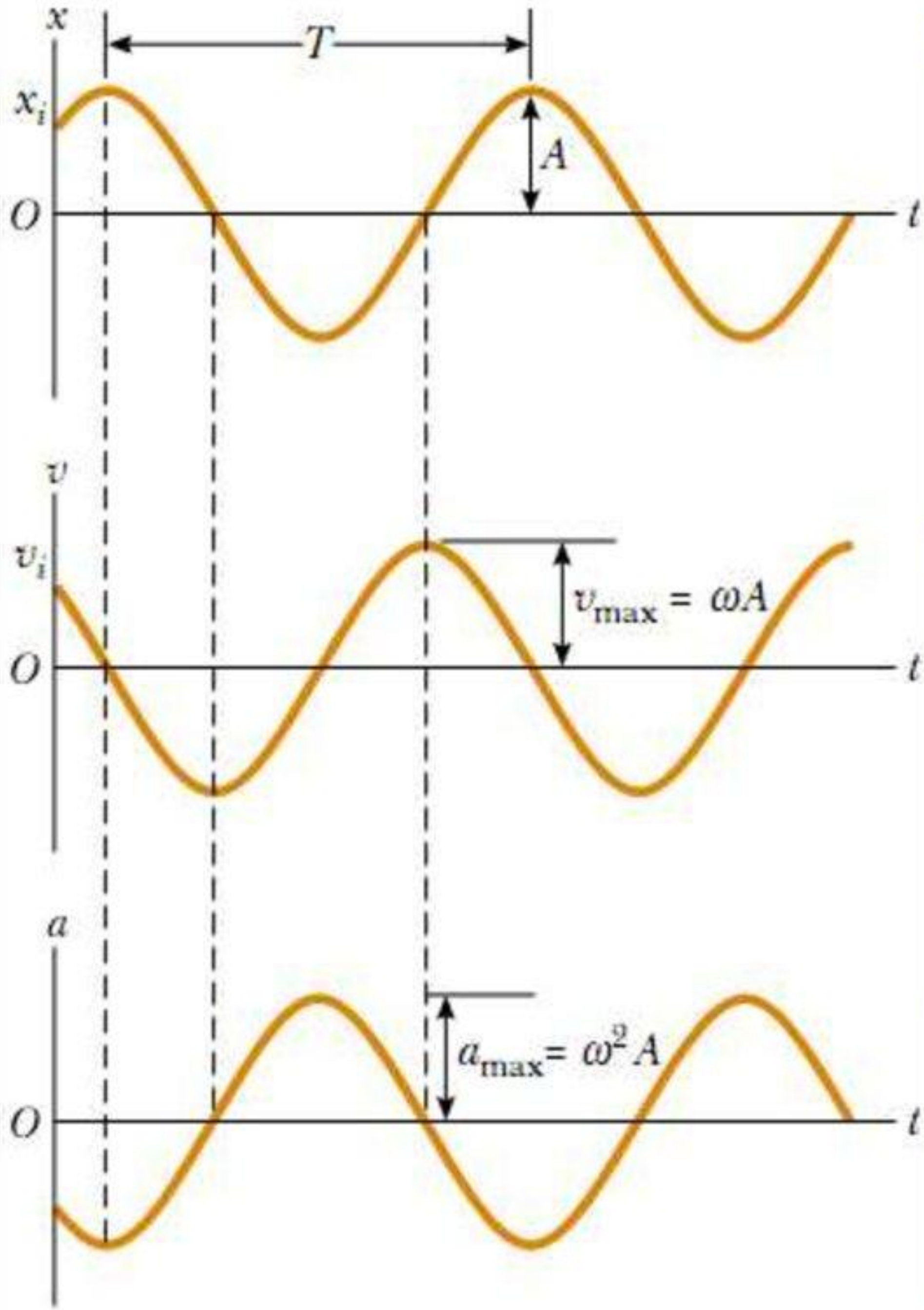
- oscillating water column
- bouncing ping-pong ball
- basic examples
- crazy examples


v
a
v
a = 0
v
a


G
F
E
D
C
B
A


Position (m)
0
Time (s)
A
B
C
D
E
F
G


Velocity (m/s)
+
0
-
Time (s)
A
B
C
D
E
F
G



- Position vs time
- Velocity vs time

  At any specified time the velocity is 90° out of phase with the position.
- Acceleration vs time

  At any specified time the acceleration is 180° out of phase with the position.

Describing oscillations

The system goes through a recurring cycle.

Amplitude, *A*, is the maximum displacement.

If the cycle repeat time is independent of amplitude (the oscillations is isochronous), then you can define a periodic time, *T*, and frequency, *f*.

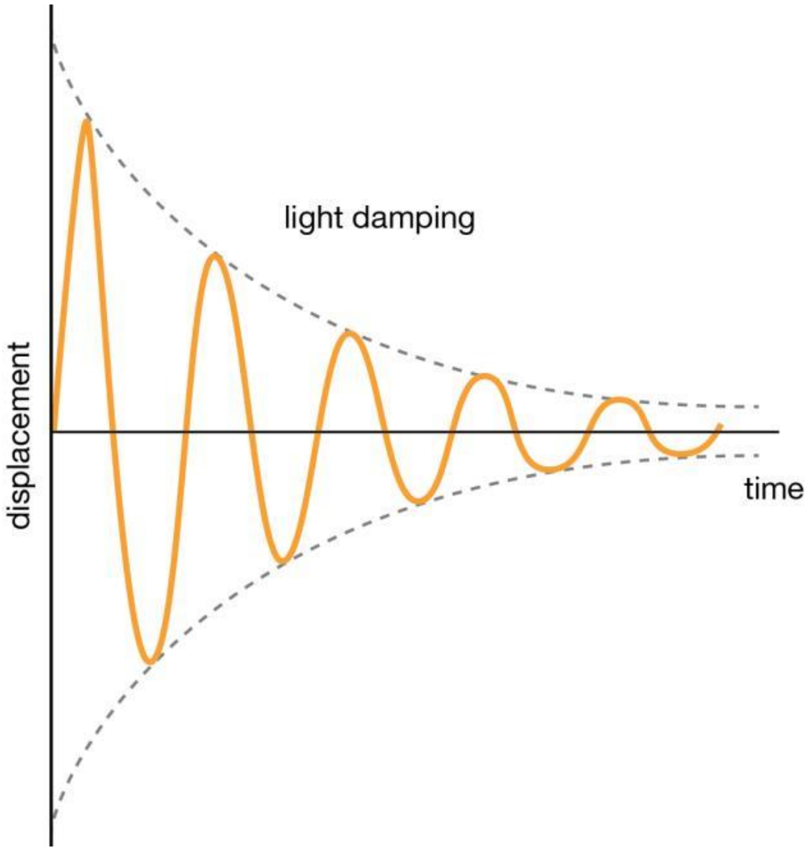
$$T = \frac{1}{f}, f = \frac{1}{T}$$

A restoring force tries to return system to equilibrium.

The system has inertia and overshoots equilibrium position.

Damping

As a system loses energy, the amplitude falls.



IOP Institute of Physics

Trolley between springs

$$F = ma = -ks$$

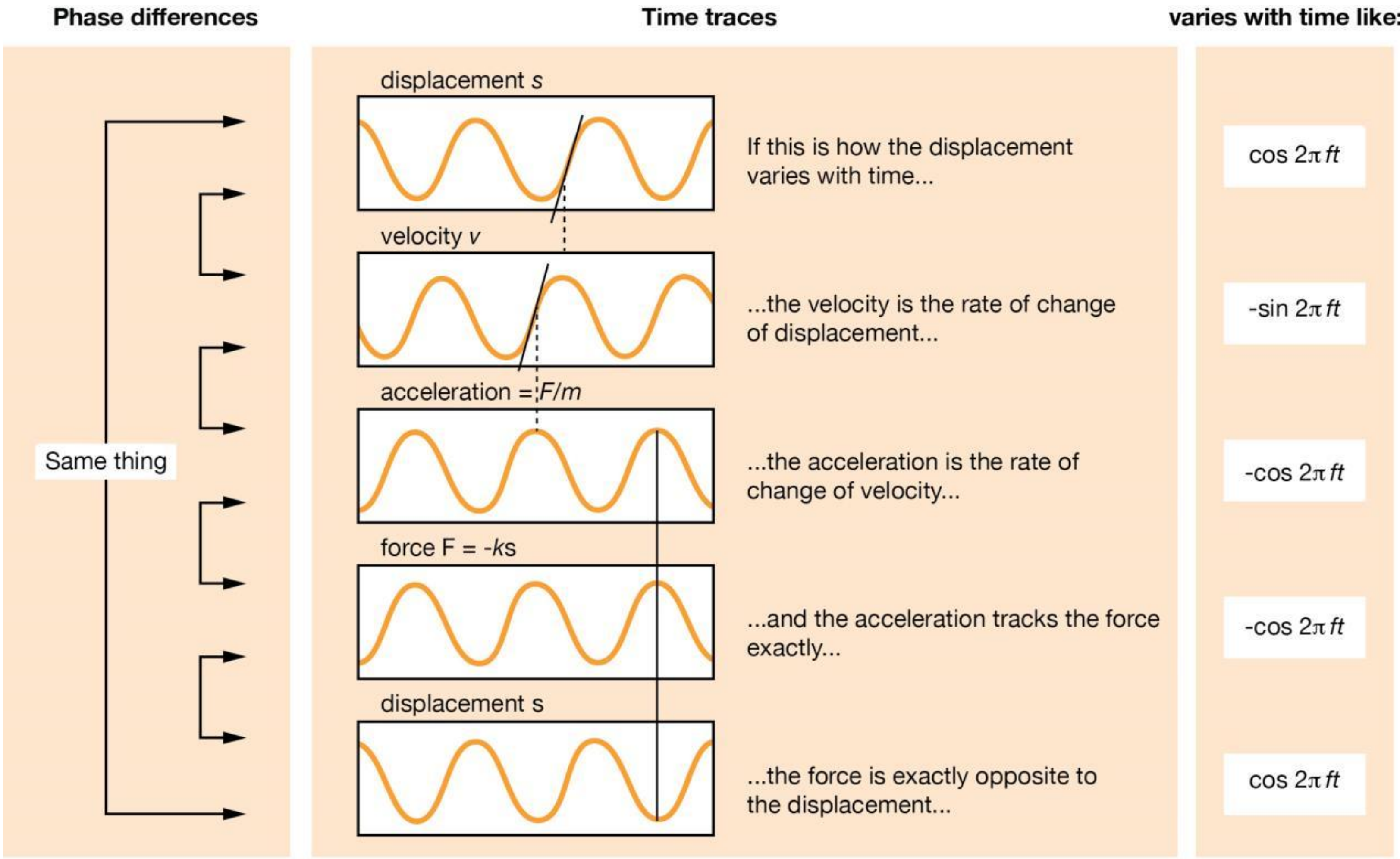
$$a = -\left(\frac{k}{m}\right)s$$

k is the constant relating restoring force to displacement, *m* is mass

The - sign indicates *a* and *s* have opposite directions.

The acceleration is

- always directed towards the equilibrium position
- proportional to displacement



Graphs of displacement, velocity, acceleration and force against time have similar shapes but differ in phase.

The SHM auxiliary circle

An imaginary circular motion gives a mathematical insight into SHM. Its angular velocity is ω.

Advanced Example Video

Basic Example Video

The time period of the motion, $T = \dfrac{2\pi}{\omega}$.

The frequency of the motion, $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$.

Displacement of the SHM, $s = A\cos(\omega t)$.

Maximum values of quantities

Simple harmonic motion:

maximum displacement = A

maximum velocity = $A\omega = 2\pi f A$

maximum acceleration = $A\omega^2 = (2\pi f)^2 A$

Circular motion:

displacement = r

velocity = $r\omega$

acceleration = $r\omega^2$

Two particular systems

Mass-on-spring, $T = 2\pi\sqrt{\frac{m}{k}}$

Simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$

(for small angle oscillations)

STOP

Finding a spring constant

Method A

Gradually load the spring with weights, and find the extension ($x = l - l_0$) at each load. Do not go beyond the elastic limit!

$F = kx$, so plot F against x and find k from the gradient.

Method B

Time 10 oscillations for a range of masses. Work out the mean period of oscillation at each mass.

$T = 2\pi\sqrt{\frac{m}{k}}$, so plot T^2 against m and find k from the gradient.

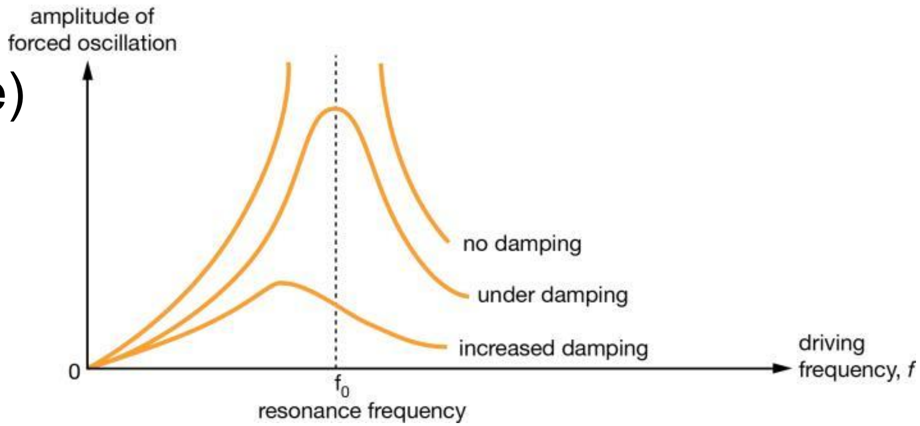
Did you get the same result both ways?
Which method do you prefer?

Forced vibrations & resonance

Millenium Bridge video clip

Amplitude of response depends on 3 factors:

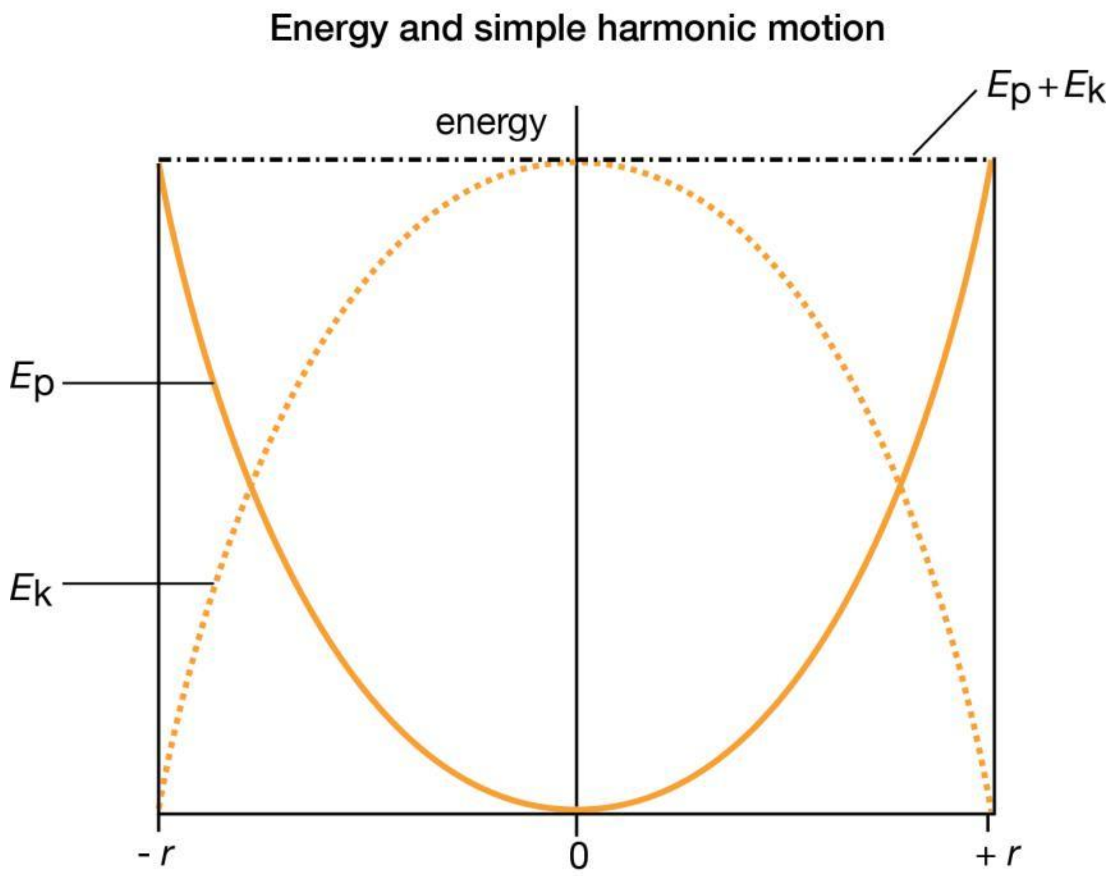
- natural frequency of vibration
- driving frequency (periodic force)
- amount of damping



Experiments:

- shaking a metre ruler
- shaking some keys
- *Advancing Physics* experiment *Resonance of a mass on a spring*
- YouTube *synchronisation*

simulation *The pendulum driven by a periodic force*



The kinetic energy of a vibrating object = ½ mv^2. The maximum kinetic energy = ½ mv_{max}^2 = ½ $m\omega^2A^2$ (since $v_{max} = \omega A$).

This makes clear that the energy of an oscillator is proportional to the square of its amplitude, A.

Problems session 2

In order of difficulty:

AP *Quick Check* Q 5,6

Practice in Physics questions on SHM

AP *Oscillator energy and resonance*

Energy and pendulums TAP 305-5

Endpoints

Video clip: The Tacoma Narrows bridge disaster

Puget Sound, Washington state, USA, November 1940