Standing waves

A special wave is formed when two ordinary identical waves travelling in opposite directions meet. The result is a standing (stationary) wave: a wave in which the crests do not move.

Objectives

By the end of this chapter you should be able to:

- state the differences between a *standing* wave and a *travelling* wave;
- describe how *a standing wave is formed*;
- draw the *various harmonics on strings and tubes* and find the *wavelength in terms of the string or tube length*;
- state the meaning of the terms *fundamental* and *harmonics*;
- state the meaning of the term *resonance*;
- solve problems with *standing waves*.

Standing waves on strings and tubes

▶ When two waves of the same speed and wavelength and equal or almost equal amplitudes travelling in opposite directions meet, a standing wave is formed. This interesting wave is the result of the superposition of the two waves travelling in opposite directions.

The main difference between a standing wave and a travelling wave is that in the former no energy or momentum is transferred. A standing wave is characterized by having a number of points at which the displacement is *always* zero. These are called nodes. (In a travelling wave, there are no points where the displacement is *always* zero.) The points at which the displacement is a maximum are called antinodes. (Note that the nodes always have zero displacement whereas the antinodes are at maximum displacement for an instant of time only.) In Figure 6.1 a string of length L has been plucked in the middle and is about to be released.

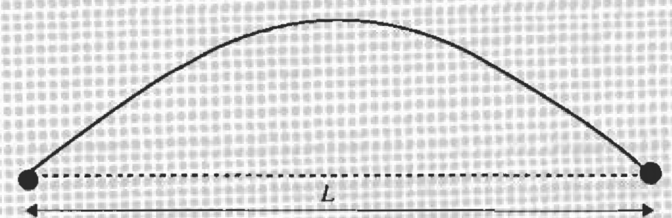


Figure 6.1 A standing wave on a string with both ends fixed. The string is held in this position and then released. A standing wave like this with a single antinode is known as a fundamental standing wave.

Successive pictures of the string will then look like Figure 6.2: the end points of the string remain fixed at all times (nodes) but the rest of the string oscillates. The middle point is the point on the string with the largest displacement (antinode). The string will return to its original position after a time equal to the period of the wave. In the absence of friction,

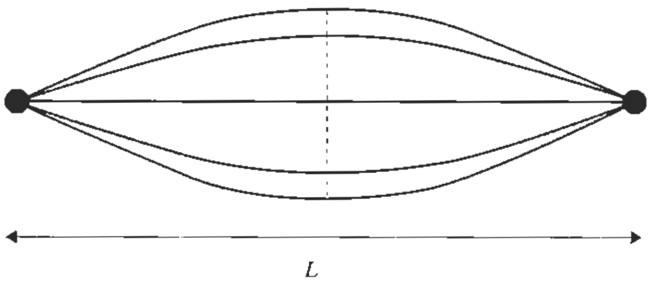


Figure 6.2 Positions of the string at various time intervals after being released. The dark circles show the positions of the nodes. The dotted line shows the position of the antinode.

this oscillation will continue forever. When the string is in its original position ($t = 0$) all the energy of the wave is in the form of potential energy of the stretched string. When the string assumes its undisturbed position, all the energy is in the form of kinetic energy. At all other positions the energy of the string consists of both potential and kinetic energy. Note that the crest of this wave (i.e. the antinode) does not move to the right or left as a crest does in a travelling wave.

The standing wave depicted above has a specific wavelength. Note that we have fitted half a full wave on the length of the string. This means that

$$\frac{\lambda}{2} = L$$
$$\Rightarrow \lambda = 2L$$

The wave with $\lambda = 2L$ is not the only standing wave that can exist on this string, however. Figure 6.3 shows the next standing wave. Note that the only constraint we have is that the ends of the string are nodes. Here, we have fitted one full wave on the string. Thus, $\lambda = L$. This standing wave has three nodes and two antinodes.

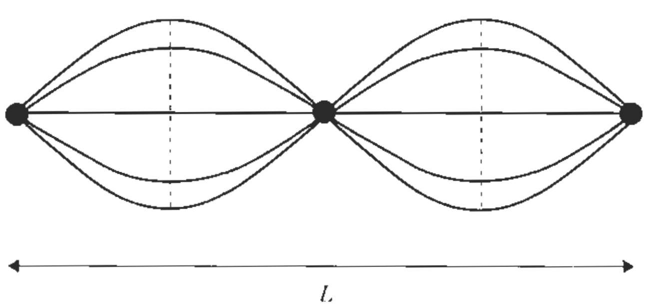


Figure 6.3 A standing wave with three nodes and two antinodes. A standing wave like this is known as the second harmonic.

An infinity of standing waves can thus exist on the string by 'fitting' waves with the constraint that the ends are nodes. The next standing wave is shown in Figure 6.4.

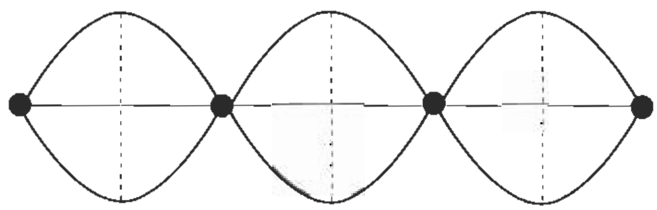


Figure 6.4 A standing wave with four nodes and three antinodes. A standing wave like this is known as the third harmonic.

For the third harmonic, we have fitted one and a half full waves on the string. Thus,

$$\frac{3}{2}\lambda = L$$
$$\Rightarrow \lambda = \frac{2L}{3}$$

In general, we find that the wavelengths satisfy

$$\lambda = \frac{2L}{n}, \quad n = 1, 2, 3, 4, \ldots$$

The wave with wavelength corresponding to $n = 1$ is called the fundamental mode of the string or the first harmonic. All other modes are called higher harmonics. So, for example, the mode with $n = 3$ is the third harmonic. The fundamental mode has the largest wavelength and thus the smallest frequency ($f = \frac{v}{\lambda}$, where v is the speed of the wave).

If f_0 is the fundamental's frequency, then all other harmonics have frequencies that are integral multiples of f_0.

Note that the distance between two successive nodes is half a wavelength. The same is true for successive antinodes. The distance between a node and the next antinode is a quarter of a wavelength.

Figure 6.5 shows that particles between two consecutive nodes move in the same direction. Particles between the adjacent pair of nodes move in the opposite direction.

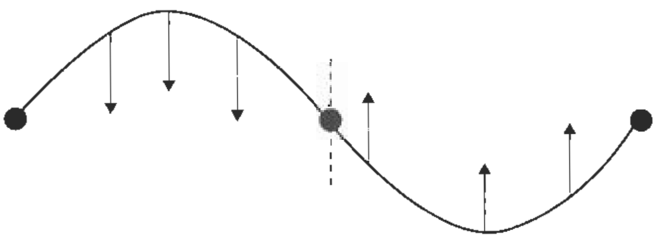


Figure 6.5 All points between two consecutive nodes are in phase: that is to say, they move in the same direction. They differ in phase by 180 with those between the next pair of nodes, which are moving in the opposite way.

If one end of the string is free and the other fixed, then the free end must be an antinode and the fixed end a node. The allowed wavelengths are then

$$\lambda = \frac{4L}{n}, \quad n = 1, 3, 5, \ldots$$

(Here n is an odd integer.) Examples of these standing waves are shown in Figures 6.6–6.8.

You must convince yourself that the wavelengths of these harmonics are indeed those given by the formula $\lambda = \frac{4L}{n}, n = 1, 3, 5, \ldots$

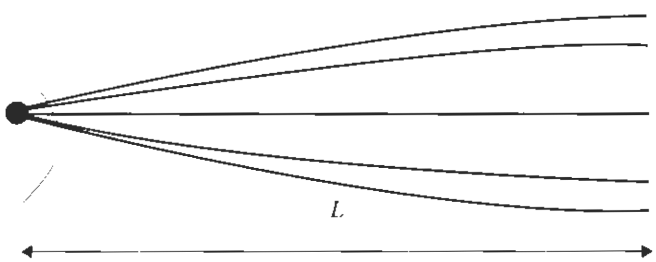


Figure 6.6 The fundamental standing wave on a string with one end fixed and the other free.

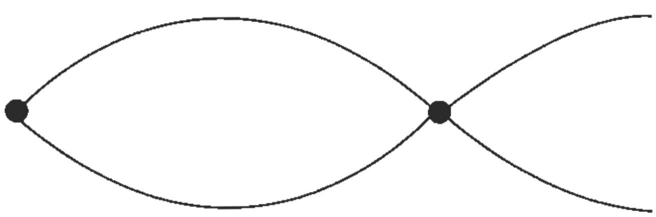


Figure 6.7 The second harmonic.

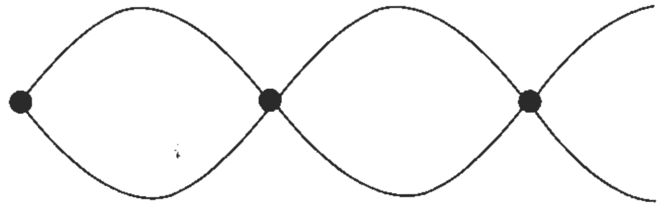


Figure 6.8 The third harmonic.

When both ends are free, the condition is

$$\lambda = \frac{2L}{n}, \quad n = 1, 2, 3, 4, \ldots$$

The situation here is entirely analogous to that with both ends fixed with the roles of node and antinode interchanged (see Figure 6.9).

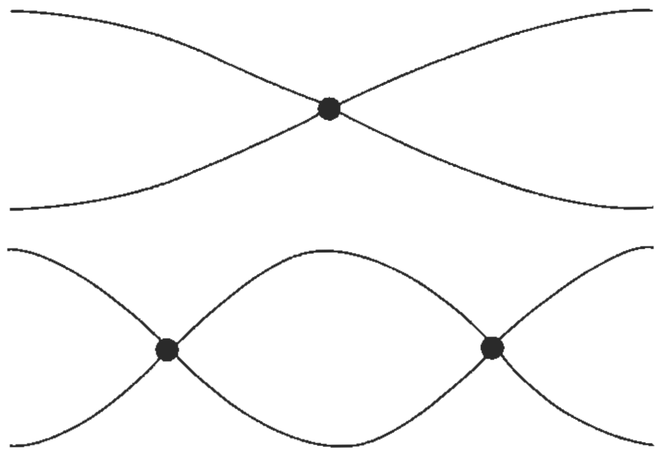
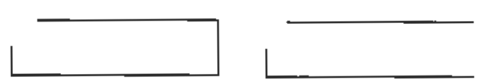


Figure 6.9 Standing waves on a string with both ends free are similar to those for both ends fixed except that nodes and antinodes are interchanged. The fundamental and second harmonic are shown here.

We have discussed standing waves exclusively in terms of waves on a string whose ends are fixed or free. Exactly the same results apply to sound standing waves formed in a pipe (such as a musical instrument) whose ends are open (corresponding to free string ends) or closed (corresponding to fixed string ends) – see Figure 6.10. Nodes exist at closed ends and antinodes at open ends.



(a) left end (mouth) is open, right end is closed

(b) left end (mouth) is open, right end is open

Figure 6.10 (a) A pipe with one end closed and one open. (b) A pipe with both ends open.

Supplementary material

Nodes in this case correspond to points in the pipe where the air molecules are not moving whereas antinodes correspond to points where the air molecules move with maximum displacement (see Figure 6.11). These are called displacement nodes and antinodes. Note, however, that at a displacement node the pressure of the gas varies the most (i.e. we have a pressure antinode), and at a displacement antinode the pressure variation is zero (i.e. we have a pressure node).

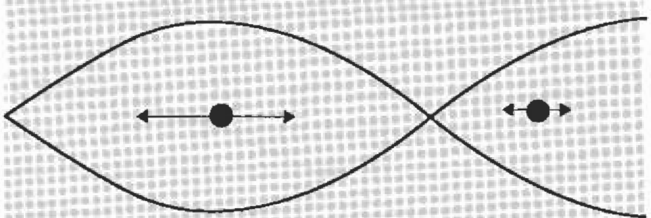


Figure 6.11 Air molecules in the pipe vibrate the most at antinodes and not at all at nodes.

You don't need to memorize the formulae for wavelength in terms of string or tube length. Rather, you should note that in all cases the distance between successive nodes or antinodes is half a wavelength and that the distance between a node and the next antinode is a quarter of a wavelength. This should allow you to figure out what kind of standing wave you can fit in the particular case you are examining. We see from these formulae that, as the length of the tube becomes smaller, the allowed wavelengths also get smaller, which means that the corresponding frequencies get larger. This is seen when you put a bottle under a tap and start to fill it with water. The falling water excites a standing wave in the bottle whose length of air column is getting smaller as the bottle fills. This means that the frequency of the sound emitted by the bottle becomes high pitched, as we know from experience.

Example questions

Q1

A standing wave is set up on a string kept under tension T. What must be done to the tension in order to double the fundamental frequency of the wave?

Answer

Since $f = \frac{v}{\lambda}$, and the wavelength is fixed in terms of the length of the string $\lambda = 2L$, we can double f by doubling the velocity of the wave. This means that the tension must increase by 4.

Q2

What is the ratio of the frequencies of the fundamental to the second harmonic for a standing wave set up on a string, both ends of which are kept fixed?

Answer

The frequencies are

$$f_0 = \frac{v}{2L} \quad \text{and} \quad f_1 = \frac{v}{L}$$

hence

$$\frac{f_0}{f_1} = \frac{1}{2}$$

Q3

A tube has one end open and the other closed. What is the ratio of the wavelengths of the fundamental to the second harmonic?

Answer

The fundamental and second harmonic have wavelengths

$$\lambda_0 = 4L \quad \text{and} \quad \lambda_1 = \frac{4L}{3}$$

hence

$$\frac{\lambda_0}{\lambda_1} = 3$$

Q4

A standing wave is set up in a tube with both ends open. The frequency of the fundamental is 300 Hz. What is the length of the tube? Take the speed of sound to be 340 m s^{-1}.

Answer

The wavelength is

$$\frac{340}{300}\,\text{m} = 1.13\,\text{m}$$

The fundamental's wavelength is equal to $2L$ and so $L = 0.57$ m.

Resonance and the speed of sound

When a vibrating tuning fork is brought near to the end of a long tube partially filled with water, a buzzing sound may be heard from the tube. When that happens, addition of more water in the tube will ruin the effect. This is an example of resonance. The tuning fork will excite the air in the tube and force it to vibrate with a frequency equal to the tuning fork's frequency. The amplitude of this standing wave will be appreciable, though, only if the frequency of the standing wave that the tube can support is equal to the tuning fork's frequency. When these two frequencies are the same, we hear the buzzing sound from the tube. Pouring more water in the tube changes the frequency of the tube and so the amplitude is now very small – no sound is heard from the tube.

This actually provides a simple method for measuring the speed of sound in air. A set of tuning forks of known frequencies are each sounded over a column of air in a long tube partially filled with water. The height of the column of water is adjusted (by pouring water in or out) until resonance is obtained (i.e. the tube emits a sound). The corresponding height of the air column and the frequency are recorded and this is repeated with the other tuning forks. The standing wave inside the tube must have a wavelength such that $\lambda = 4L$, where L is the length of the air column. But $\lambda = \frac{v}{f}$, where f is the corresponding frequency, which equals the known frequency of the tuning fork. Thus, v, which is the speed of sound, can be determined by repeating this procedure for various different tuning forks and then plotting L versus $1/f$. One must get a straight line with slope $v/4$.

Supplementary material

This discussion ignores end corrections. End corrections are necessary in practice because the standing wave may have a wavelength that does not satisfy $\frac{\lambda}{4} = L$ but rather $\frac{\lambda}{4} = L + e$, where e is a constant depending on the diameter of the tube. In an experiment to measure the speed of sound by resonance the end correction *must* be included.

Resonance is a general phenomenon. It occurs whenever a system that is capable of oscillation or vibration is subjected to an external disturbance with a frequency equal to the natural frequency of the system itself. In that case, the system oscillates with a large amplitude. If the frequencies do not match, the system still vibrates but the amplitude is very small. Clearly, resonance can be a dangerous phenomenon. A system that is set into vibration by something external and develops large amplitudes may eventually break or fall apart. Aeroplane wings, engines, bridges, tall buildings, etc., must all be protected against resonance from external vibrations due to wind, other vibrating objects, etc. Soldiers always break their step when walking over a bridge, in case the force that they exert on the bridge starts uncontrollable oscillations of the bridge. An earthquake may set a building into oscillation if the frequency of the longitudinal wave created by the earthquake is equal to the natural frequency of vibration of the building. This frequency is $\frac{c}{2L}$, where c is the speed of sound in the structure of the building and L is its height. (See Figures 6.12–6.14.)

Figure 6.12 The Tacoma Narrows bridge collapsed in 1940, a victim of resonant failure.

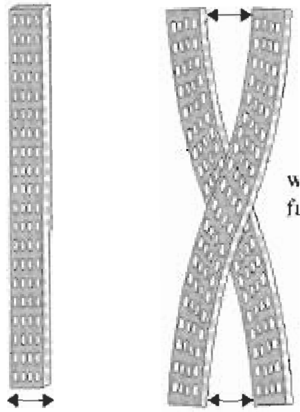


Figure 6.13 A building will be made to oscillate in a standing wave mode if the frequency of the earthquake wave matches the natural frequency of oscillation of the building.

Figure 6.14 The severe earthquake that struck northern Turkey in August 1999 released vast amounts of energy. Hundreds of buildings toppled and tens of thousands of people were killed.

Questions

1 Describe what is meant by a standing wave. In what ways does a standing wave differ from a travelling wave?

2 How is a standing wave formed?

3 In the context of standing waves describe what is meant by:
(a) node;
(b) antinode.

4 Describe how you would arrange for a string that is kept under tension, with both ends fixed, to vibrate in its second harmonic mode. Draw the shape of the string when it is vibrating in its second harmonic mode.

5 Explain what is meant by resonance and give two examples where it occurs.

6 Car drivers occasionally experience a 'shaking steering wheel' when travelling at a particular speed. The shaking disappears at lower or higher speeds. Suggest a reason for this observation.

7 A string is held under tension, with both ends fixed, and has a fundamental frequency of 250 Hz. If the tension is doubled, what will the new frequency of the fundamental mode be?

8 A string has both ends fixed. What is the ratio of the frequencies of the first to the second harmonic?

9 The fundamental mode on a string with both ends fixed is 500 Hz. What will the frequency become if the tension in the string is increased by 20%?

10 The wave velocity of a transverse wave on a string of length 0.500 m is 225 m s^{-1}.
(a) What is the fundamental frequency of a standing wave on this string if both ends are kept fixed?
(b) While this string is vibrating in the fundamental harmonic, what is the wavelength of sound produced in air?
(Take the speed of sound in air to be 330 m s^{-1}.)

11 Figure 6.15 shows a tube with one end open and the other closed. Draw the standing wave representing the third harmonic standing wave in this tube.

Figure 6.15 For question 11.

12 A glass tube is closed at one end. The air column it contains has a length that can be varied between 0.50 m and 1.50 m. If a tuning fork of frequency 306 Hz is sounded at the top of the tube, at which lengths of the air

column would resonance occur? (Take the speed of sound to be 330 m s^{-1}.)

13 A glass tube with one end open and the other closed is used in a resonance experiment to determine the speed of sound. A tuning fork of frequency 427 Hz is used and resonance is observed for air column lengths equal to 17.4 cm and 55.0 cm.

(a) What speed of sound does this experiment give?

(b) What is the end correction for this tube?

14 A tube with both ends open has two consecutive harmonics of frequency 300 Hz and 360 Hz.

(a) What is the length of the tube?

(b) What are the harmonics?

(Take the speed of sound to be 330 m s^{-1}.)

15 A string of length 0.50 m is kept under a tension of 90.0 N and vibrates in its fundamental mode. The mass of the string is 3.0 g.

(a) What is the frequency of the sound emitted? (Take the speed of sound to be 330 m s^{-1}.)

(b) The same string now vibrates in water. What is the wavelength of the sound emitted? (Take the speed of sound in water to be 1500 m s^{-1}.)

16 A container of water of length 12 cm is placed on top of a vibration generator (Figure 6.16). When the generator is turned on, the water in the container sloshes back and forth.

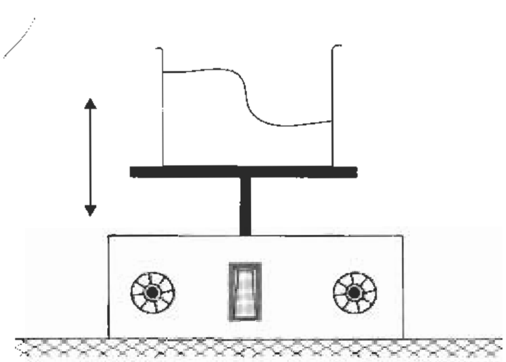


Figure 6.16 For question 16.

When the frequency is adjusted to about 0.75 Hz, the water actually spills out of the container.

(a) Suggest a reason for this.

(b) Estimate the speed of water waves in the container.

17 Do the following experiment at home. Take a styrofoam cup (top diameter approximately 8 cm) and fill it with cold coffee or tea. Now drag it slowly over a surface that is neither too smooth nor too rough, for example a kitchen counter.

(a) Observe and explain what you see on the surface of the liquid as the speed at which you drag the cup is varied.

(b) Knowing that the speed of water waves in the cup is about 15 m s^{-1}, estimate the frequency that makes the water vibrate.

(c) Is this frequency related to the speed of the cup?

18 Consider a string with both ends fixed. A standing wave in the second harmonic mode is established on the string, as shown in Figure 6.17. The speed of the wave is 180 m s^{-1}.

(a) Explain the meaning of wave speed in the context of standing waves.

(b) Consider the vibrations of two points on the string, P and Q. The displacement of point P is given by the equation $y = 5.0 \cos(45\pi t)$, where y is in mm and t is in seconds. Calculate the length of the string.

(c) State the phase difference between the oscillation of point P and that of point Q. Hence write down the equation giving the displacement of point Q.

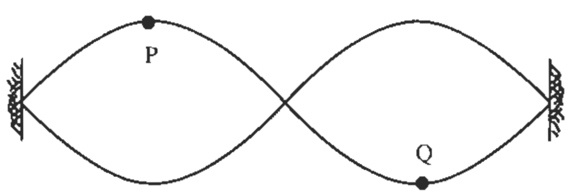


Figure 6.17 For question 18.

19 A sound wave of wavelength 1.7 m passes through air, where the speed of sound is 330 m s^{-1}. Assume that a molecule of air has mass 4.8×10^{-26} kg and that, as a result of the sound wave, it oscillates with an amplitude of 4.0×10^{-7} m. Calculate the maximum kinetic energy of the molecule due to its oscillations.

20 A string with both ends fixed vibrates in the third harmonic mode, as shown in Figure 6.18. The length of the string is 6.0 m and the speed of the wave is 120 m s^{-1}.

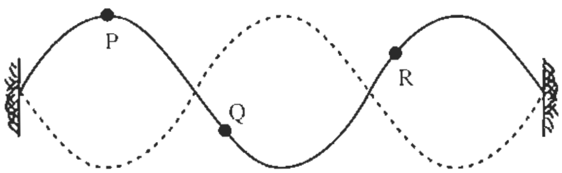


Figure 6.18 For question 20.

(a) Calculate the wavelength of the wave on the string.

(b) The amplitude of oscillation of point P is 4.0 mm. Explain why the displacement of point P is given by the equation $y = 4.0\cos(60\pi t)$, where y is in millimetres and t is in seconds.

(c) The amplitude of oscillation of points Q and R is 2.0 mm. State the equation giving the displacement of (i) point Q and (ii) point R.

(d) Calculate the average speed of (i) point P and (ii) point Q from to $t = 0$ to $t = \frac{T}{4}$, where T is the period of the wave.

(e) Calculate the maximum speed of (i) point P and (ii) point Q.