~~EARS 5~~

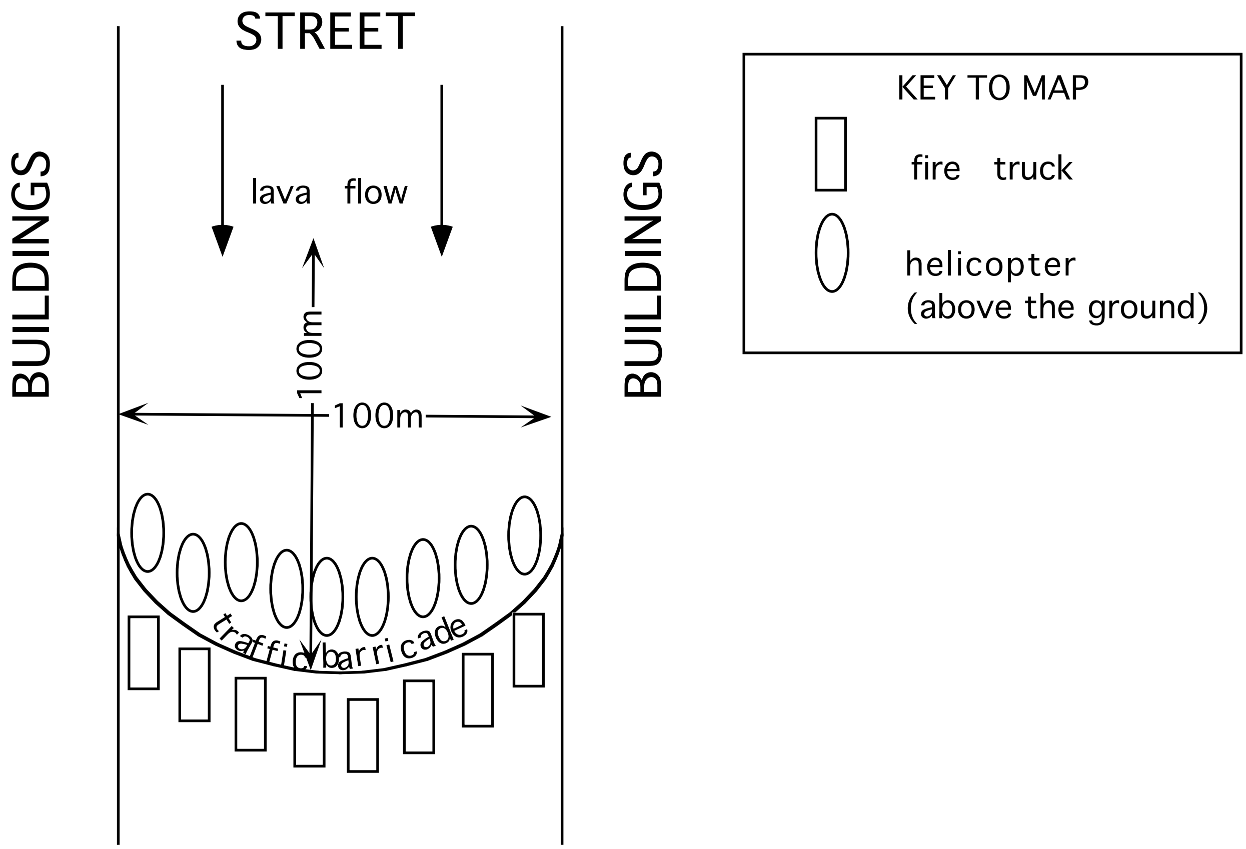
Natural disasters and catastrophes

~~Assignment #2~~

Magmatic crystallization, latent heat, and specific heat

~~Due Wednesday July 30~~

In the blockbuster movie *Volcano*, a volcano erupts in the heart of Los Angeles; pahoehoe lava pours down the streets of the city, destroying buildings, starting fires, and causing general panic. A climactic scene of the film portrays firefighters and helicopter pilots stopping the main lava flow by damming it behind a traffic barricade and spraying large quantities of water on it, thereby cooling and solidifying it in a matter of a minute or two. A map-view sketch of the situation is shown below.



<u>Assume that:</u>

1) The lava flow needs to solidify to a depth of 0.75 m over a surface area of 100 m x 100 m in order to create a crust sufficiently strong and extensive to resist the pressure from upstream of the still molten and flowing magma.

2) Once the water sprayed on the magma has turned to steam, it escapes, so cannot further absorb heat from the magma.

3) Densities of magma and solid basalt are approximately the same. (They actually differ by roughly 10%, but since your calculation is not going to give you an answer that precise, it doesn't matter.)

<u>Your job:</u>

Assess the physical plausibility of the scene. This involves two parts:

1. Design and carry out back-of-the envelope calculations designed to answer questions a and b. Be sure to state all assumptions that you make in your calculations. Hint: calculate the total amount of heat the magma must lose in order to cool and crystallize, then calculate the mass of water needed to absorb this energy by conversion to steam.

a) Could the fire trucks and helicopters provide, from their tanks, a sufficient volume of water to stop the flow?

b) If, instead of being taken from the internal tanks of helicopters and fire trucks, the water were pumped from city water mains by the 15 trucks, how long would it take to stop the flow? Assume each truck can connect to a different hydrant, so that each can pump at a 2000 gal/min rate.

2. List and discuss at least two ways in which your back-of-the-envelope calculations might give *inaccurate* (note: not *imprecise*!) answers. In other words, what assumptions might be in error, or what physical processes might be important but neglected, in your calculation? How might you quantitatively take these into account in a revised calculation?

Keep in mind that a calculation can convey an argument equally well as can prose, so you will want to set up your calculations with as much care and clarity as you would a prose argument in a paper. Be careful about the use of significant figures. As on the previous assignments, you will be graded both on the correctness of your calculation and use of significant figures, as well as the quality of your argument and discussion.

Please help the graders: Feel free to hand-write all math and equations, but please type as much of the rest of your answers as possible. Thanks!

Mathematical relationships

Mass = density * volume
Energy transferred by changing state = latent heat * mass
Energy transferred in changing temperature of a substance =
specific heat * temperature change * mass

Numbers that you may find useful:

Density of basalt or basaltic magma: 3.0×10^3 kg m^{-3}
Specific heat of basaltic magma: 1.0×10^3 J kg^{-1} $°C^{-1}$
Specific heat of (solid) basalt: 1.4×10^3 J kg^{-1} $°C^{-1}$
Latent heat of crystallization of basaltic magma: 4.0×10^5 J kg^{-1}
Temperature of the magma: 1350 °C
Crystallization temperature of basalt: 1200°C
Density of water: 1.0×10^3 kg m^{-3}
Specific heat of steam: 2.0×10^3 J kg^{-1} $°C^{-1}$
Specific heat of liquid water: 4.2×10^3 J kg^{-1} $°C^{-1}$
Latent heat of vaporization of water: 2.3×10^6 J kg^{-1}
Temperature of water when sprayed: 20°C
Carrying capacity of a fire truck (internal tanks): 2000 gallons
Carrying capacity of a helicopter: 5000 gallons
Pumping rate of a fire truck (if water supplied from city water main via fire hydrants): 2000 gallons/min
Number of fire trucks: 15
Number of helicopters: 20

Equivalencies

1 erg=10^{-7} J (J=Joules)
1 gallon of water = 3.8 liters = 3.8 kg

Background: *Specific heat and latent heat.* In order to change the temperature of a substance, heat must be added (to raise its temperature) or removed (to lower its temperature). However, different substances require different amounts of heat to change their temperature by the same amount. The quantity that describes this is the *specific heat*. Specifically (pun intended?), the specific heat is the energy required to change the temperature of a given mass of a substance by one degree. Thus, because it requires 4200 Joules to raise 1 kg (= 1 liter = 2.2. lbs) of water by one degree Centigrade, the specific heat of water is 4.2×10^3 J kg^{-1} $°C^{-1}$. If the substance were not water, but the same mass of rock, it would require less energy, about 1000 J, to raise the temperature by 1 degree because the specific heat of rock is less than that of water.

Expressing this concept mathematically, the energy (ΔE) involved in heating or cooling a substance is proportional to the quantity of the substance (its mass M) and the temperature change (ΔT). The constant of proportionality is the specific heat, C_p:

$$\Delta E = C_p M \Delta T$$

Energy must be added to increase the temperature, but is released when the temperature drops.

Latent heat is a concept very similar to specific heat, but instead of quantifying the heat involved in changing the temperature of a substance, it quantifies the heat needed to change the *state* of a substance, that is, to evaporate, melt, freeze, or condense it. You have undoubtedly watched a pot boiling on the stove. Energy is going into the water from the stove burner, but the temperature of the water is constant at 100°C; the energy is going into changing the state of the water from liquid to vapor (evaporating it). If all that water vapor were later to condense, an equivalent amount of energy would be released in the conversion from vapor to liquid. The latent heat is the energy required to change the state of a given mass of a substance. Mathematically, this is expressed as

$$\Delta E = LM$$

where L is the latent heat and M is the mass. Energy must be added to melt or evaporate a substance, but is released when a substance condenses or freezes.

Using the value of latent heat listed above, it requires 2.3×10^6 Joules to vaporize 1 kg (1 liter) of liquid water, so the latent heat of evaporation of water is 2.3×10^6 J kg^{-1}. Note that it takes more than 500 times as much energy to vaporize water as to change the water temperature by one degree; equivalently, you could say that it takes more than 5 times as much energy to vaporize water as it does to raise its temperature all the way from 0°C to 100°C. In thinking ahead to our discussion of weather-related disasters, keep in mind that an important source or sink of energy in the atmosphere and hydrosphere is the change of state of water.