Topic 1 – Physical Measurements

1.2b – Uncertainties and Errors

Measurement Errors

Physics is fundamentally about *measuring* the physical universe.

Whenever you actually measure something then you are always comparing it against a standard and there is always a chance that you can make an error.

Errors can be of two general types:

- Random – these are unpredictable errors brought about by things usually out of your control e.g. electrical noise affecting an ammeter reading.
- Systematic – these are errors usually brought about by the measuring equipment (or its operator) e.g. a calliper that does not actually read zero when it should.

Measurement Errors

- **Random** errors can be reduced by repeating readings.
  - As the error is random, some measurements will be high, others low but on average they should be more precise.
- **Systematic** errors can be reduced by calibrating equipment.
  - By checking zero readings and scale calibration, systematic errors can be calculated and compensated for.

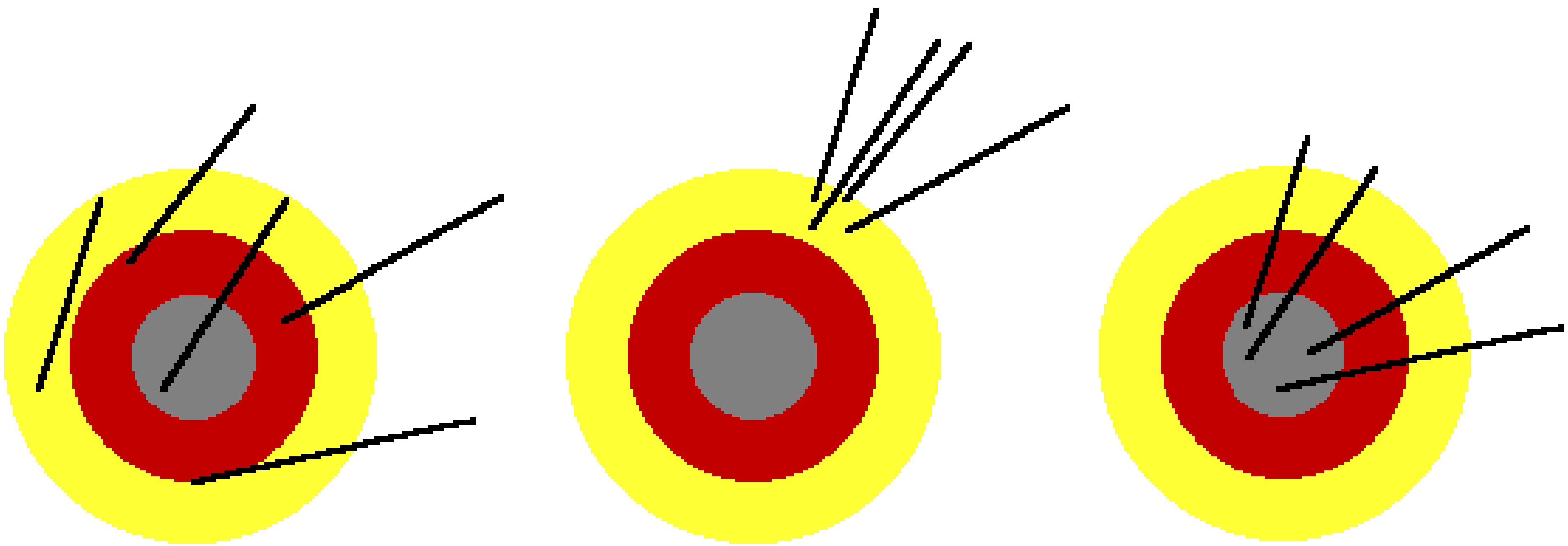
Accuracy and Precision

An experimental result is described by its **accuracy** (how close it is to the true value) and its **precision** (how close repeat readings are together)

Accuracy is improved by reducing systematic errors

Precision is improved by reducing random errors.

Accuracy and Precision



good accuracy
poor precision

poor accuracy
good precision

good accuracy
good precision

Reporting Results

To a scientist, the number of significant figures quoted in an answer indicates the precision of that answer.

Example

A value quoted as 5 kJ (1sf) suggests an answer between 4500J and 5500J. i.e. a range of 1000J or 1kJ

A value quoted as 5.00 kJ (3sf) suggests an answer between 4995 J and 5005 J i.e. a range of just 10 J or 0.01kJ

Reporting Results

The golden rule in reporting results is very simple.

The number of significant digits in a result should not exceed that of the <u>least</u> precise raw value on which it depends.

Questions:

Calculate 1.2m / 3.65s

Calculate 605N x 12m

Calculate 4.05m + 3.54 mm

Uncertainties

All physical measurements have an associated uncertainty.

This reflects the errors involved in making the measurement.

- If the error is **systematic** then the uncertainty is usually **±the smallest division** on the instrument.
  - e.g. a 30cm ruler has an uncertainty of ±1mm or ±0.1cm
- If the error is random then the uncertainty can be determined through the use of repeat readings.
  - e.g. an ohmmeter is used to determine the resistance of a component and reads values of: 1450 Ω 1490 Ω 1475 Ω
  - The error (by precision) is just ±1Ω BUT the mean of the results is 1472Ω.
  - In this case the value should be *best* stated as 1472±22Ω
    - (1472-1450 = 22 and 1490-1472 = 18 therefore largest uncertainty is 22)

Uncertainties

If the error is random and caused by a human than an estimate of the size of the error is allowed.

e.g. a stopwatch records a time as 5.46 s.

Human reaction time is approximately 0.3 s. You can check yours online if you want.

http://faculty.washington.edu/chudler/java/redgreen.html

Therefore the time is 5.46 ± 0.3 s

BUT the 0.3 is far more significant than the 6 in 5.46 so the actual result to record is:

5.5 ± 0.3 s

Notice the following:

The uncertainty is of the same precision as the measurement

The unit always follows the uncertainty not the measurement

Types of Uncertainties

The type of uncertainty seen so far is an **absolute** uncertainty.

- This is often written as Δx if the measurement is x
  - Δ (Delta) traditionally means “change in”

A **fractional** uncertainty is found by using:

$$\text{fractional uncertainty} = \frac{\text{absolute uncertainty}}{\text{absolute measurement}} = \frac{\Delta x}{x}$$

A **percentage** uncertainty is found by using:

$$\text{percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{absolute measurement}} \times 100$$

$$\Delta_{\%} x = \frac{\Delta x}{x} \times 100$$

Combining Uncertainties

When adding or subtracting measurements with uncertainties ADD the absolute uncertainties

Example

1.56 ± 0.02 m + 4.53 ± 0.05 m = 6.09 ± 0.07m

When multiplying or dividing (or using indices) measurements with uncertainties ADD the percentage uncertainties.

Example

1.56 ± 0.02 m x 4.53 ± 0.05 m becomes

1.56 ± 1.28% m x 4.53 ± 1.10% m

7.0668 ± 2.38% m^2

7.0668 ± 0.1681 m^2

7.07 ± 0.17 m^2 which *could* become 7.1 ± 0.2 m^2

Combining Uncertainties

When dealing with uncertainties related to other functions such as trigonometric functions and logarithms the uncertainty **range** is used.

example

An angle, θ, is measured as 45 ± 1°

The uncertainty in sinθ is given by:

Sin 45 = 0.707

Sin 44 = 0.694 (-0.011)

Sin 46 = 0.719 (+0.012)

Sin 45 = 0.707± 0.012 OR 0.71 ± 0.01

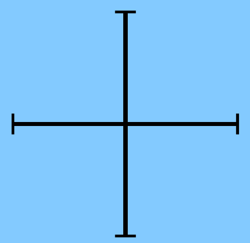
Graphing Uncertainties

The uncertainty in both the abscissa (x-value) and ordinate (y-value) of a data point to be plotted should always be calculated.

IF these uncertainties could be visible on the graph (if they are significant) then error bars **MUST** be drawn on the data points of your graph.

This is usually easy to do in a spreadsheet or graphing program.

Error bars look like:



Graphing Uncertainties

When calculating a gradient from a graph it is important to estimate the magnitude of the uncertainty in it.

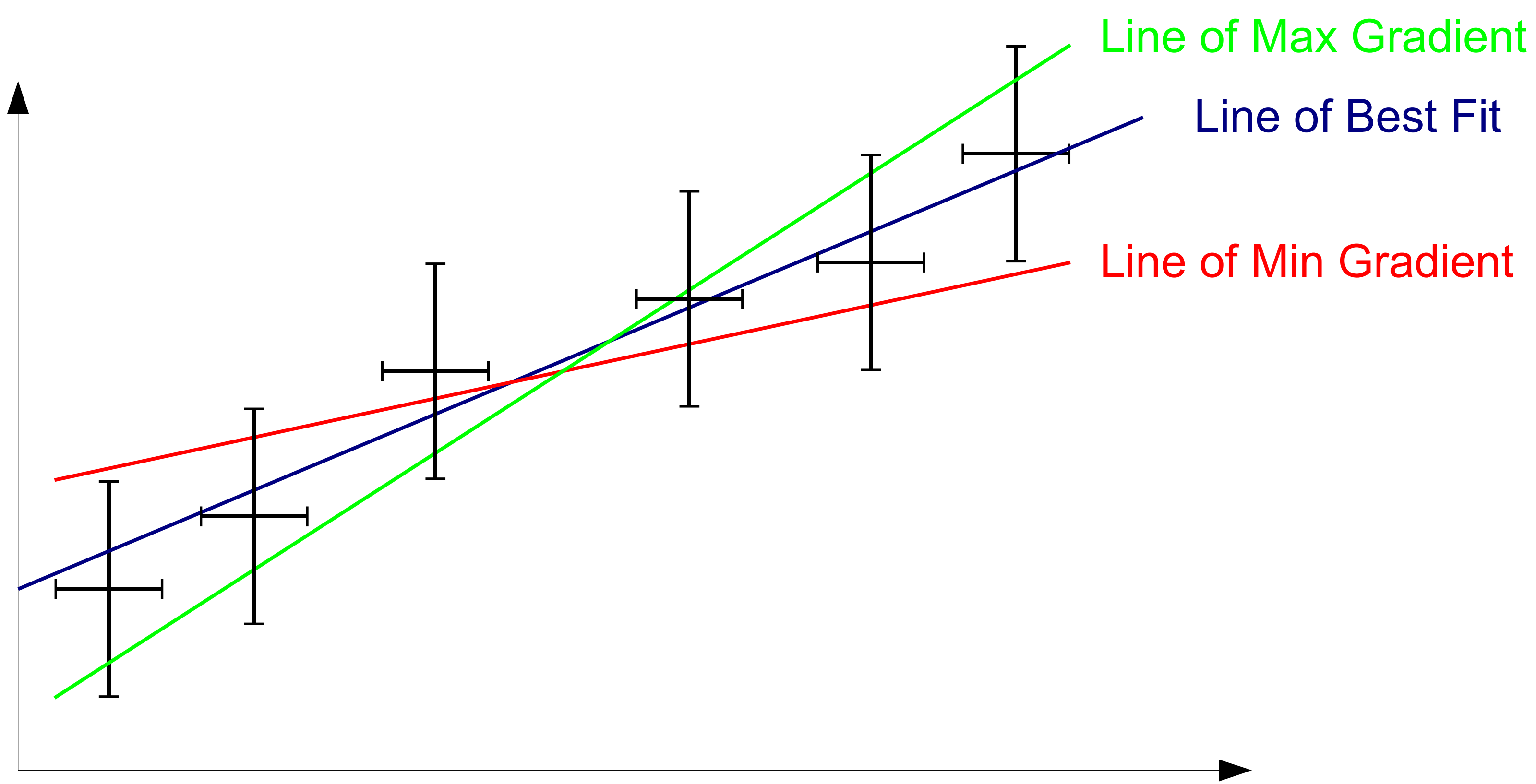
This is done by drawing onto the graph two lines of “worst fit”

- Also known as the lines of minimum and maximum gradient.

These are drawn by imagining a square drawn around the error bars of the two extreme data points

- The top left corner of data point 1 is joined to the bottom right corner of data point n
- The bottom left corner of data point 1 is joined to the top right corner of data point n
- The gradients of these two lines are then calculated and the one with the largest deviance from the line of best fit is quoted as the uncertainty.

Graphing Uncertainties



Graphing and Logarithms

Often simply plotting x versus y will not yield a straight line graph.

In order to make use of these sorts of graphs we often use logarithms. Error bars are not needed on log graphs

Assume that a graph has a curved form given by:

$y = kx^n + c$

By taking logs this can be reduced to a simple straight line

$\log (y-c) = \log (kx^n)$

Note as c is simply the y intercept it is easy to combine with the y-values

The law of logs says $\log (AB) = \log A + \log B$

So

$\log (y-c) = \log x^n + \log k$

The law of logs also says that $\log A^n = n \log A$

So

$\text{Log} (y-c) = n \log x + \log k$

i.e. a straight line graph of log x versus log (y-c) of gradient n and ordinate intercept of log k