Topic 2: Mechanics

2.1 – 2-D MOTION

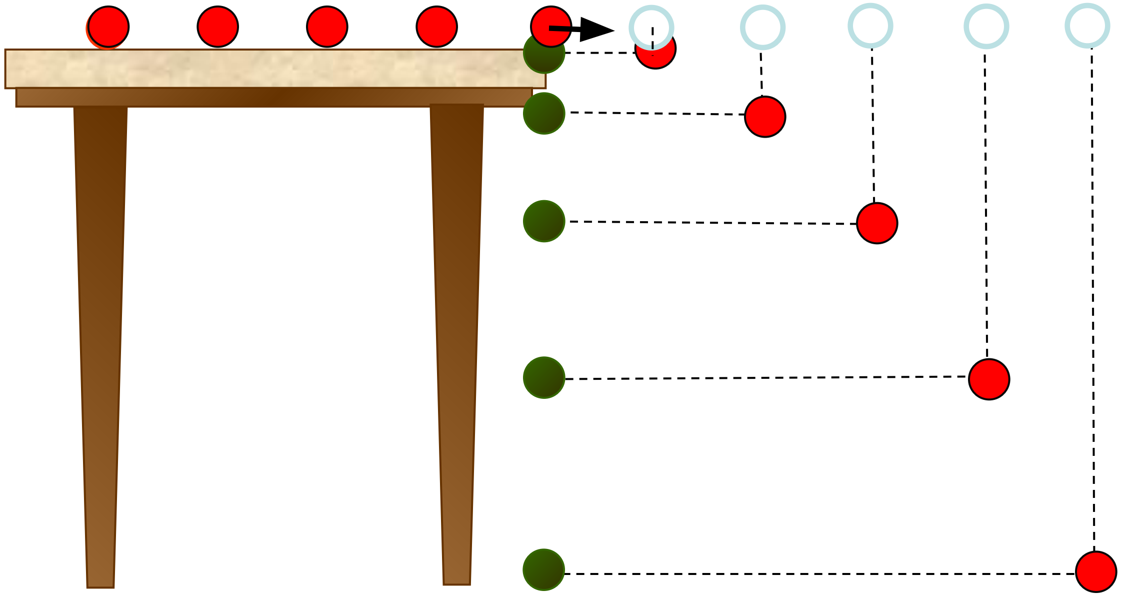
Projectile motion

- A **projectile** is an object that has been given an initial velocity by some sort of short-lived force, and then moves through the air under the influence of gravity.
- Baseballs, stones, or bullets are all examples of projectiles executing **projectile motion**.
- You know that all objects moving through air feel an air resistance (recall sticking your hand out of the window of a moving car).

- We will ignore air resistance in the discussion that follows…

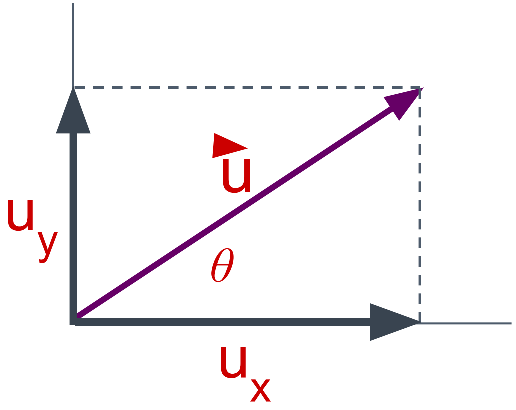
Horizontally Launched Projectiles

A ball rolling of the table is an excellent example of an object thrown into the air with horizontal initial velocity (velocity at the time when the object is launched). The ball becomes airborne when leaving the table.



black vector represents initial velocity of the horizontally launched projectile

Projectile motion



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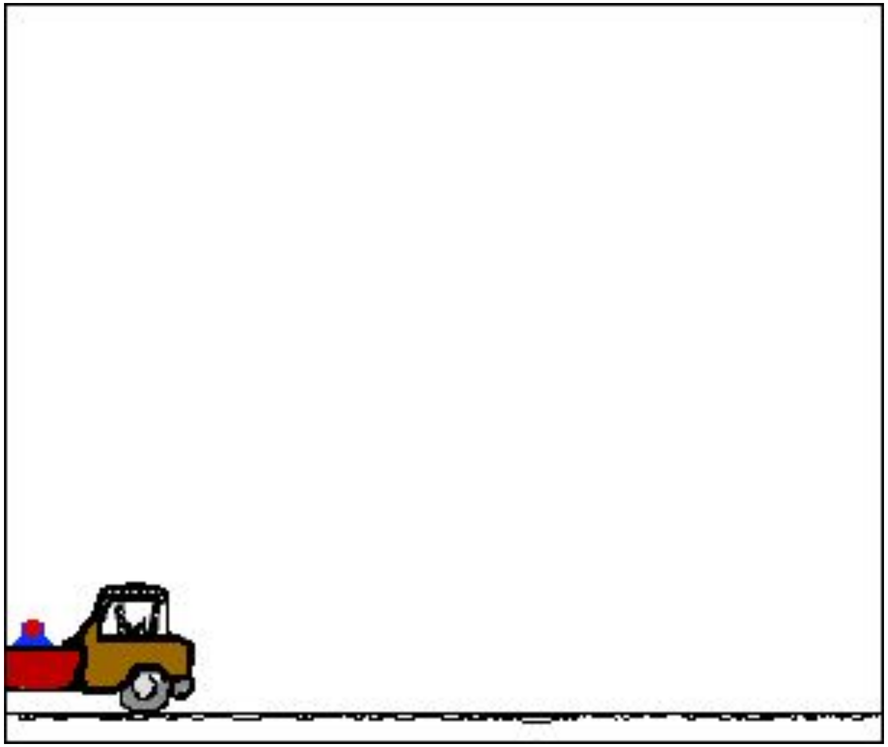
Horizontal component of motion for a projectile is completely independent of the vertical component of the motion. Their combined effects produce unique path - parabola.

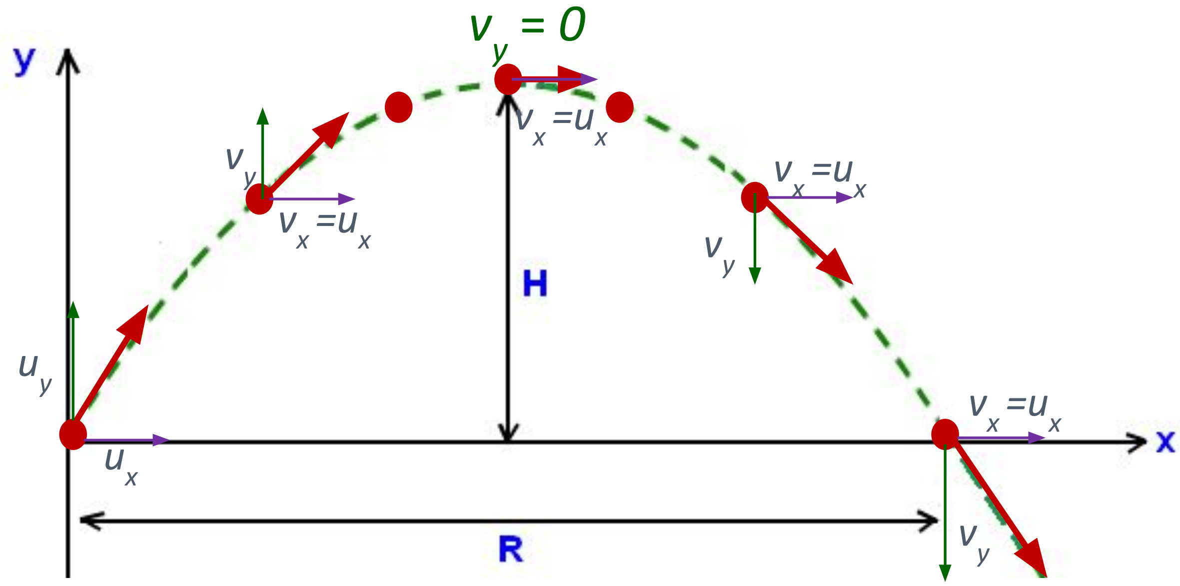
- a_x = 0 in the absence of air resistance.
- a_y = -10m/s^2 in the absence of air resistance.

The Truck and The Ball

Imagine a pickup truck moving with a constant speed along a city street. In the course of its motion, a ball is projected straight upwards by a launcher located in the bed of the truck. Imagine as well that the ball does not encounter a significant amount of air resistance. What will be the path of the ball and where will it be located with respect to the pickup truck?



Analysing projectile motion



$v_x = u_x$ $\Delta x = u_x t$

$v_y = u_y - 10t$ $\Delta y = u_y t - 5t^2$

The following slides contain practice problems

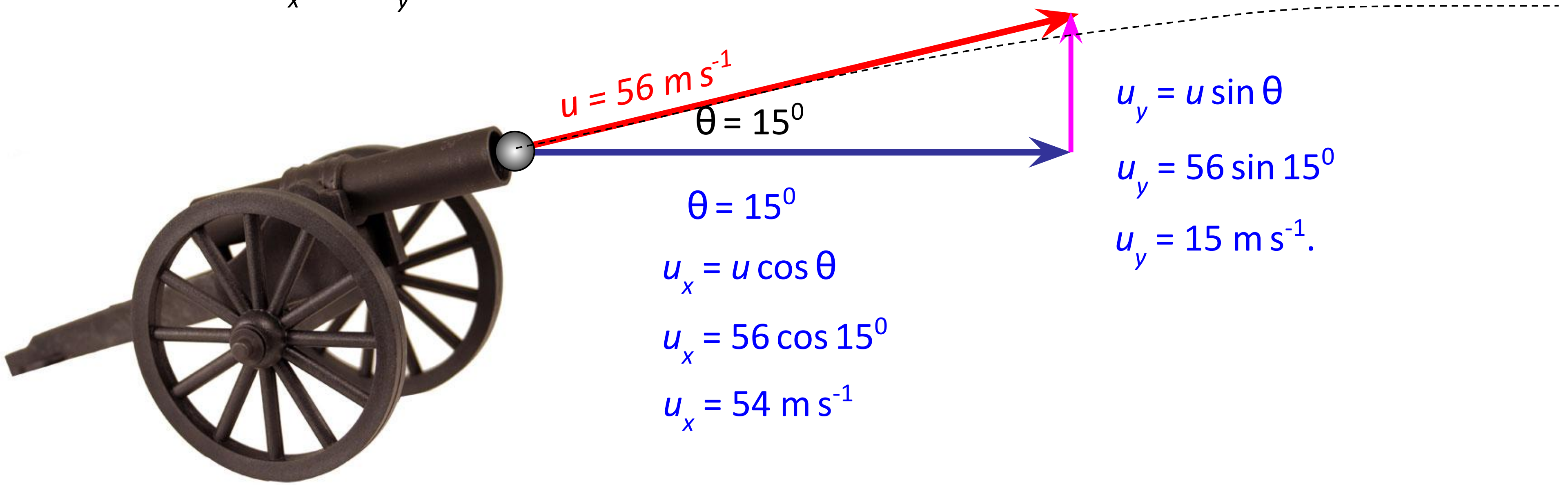
Analysing projectile motion

$$\Delta x = u_x t \qquad \Delta y = u_y t - 5t^2$$

$$v_x = u_x \qquad v_y = u_y - 10t$$

A cannon fires a projectile with a muzzle velocity of 56 ms^{-1} at an angle of inclination of 15^0.

(a) What are u_x and u_y?



$\theta = 15^0$

$u_x = u \cos \theta$

$u_x = 56 \cos 15^0$

$u_x = 54 \text{ m s}^{-1}$

$u_y = u \sin \theta$

$u_y = 56 \sin 15^0$

$u_y = 15 \text{ m s}^{-1}$.

(b) What are the tailored equations of motion?

$v_x = 54$

$\Delta x = 54t$

$v_y = 15 - 10t$

$\Delta y = 15t - 5t^2$

(c) When will the ball reach its maximum height?

At the maximum height, $v_y = 0$. Why?

$v_y = 15 - 10t \rightarrow 0 = 15 - 10t$ $\boxed{t = 1.5\text{ s}}$

(d) How far from the muzzle will the ball be when it reaches the height of the muzzle at the end of its trajectory

From symmetry $t_{up} = t_{down} = 1.5$ s so $t = 3.0$ s.

$\Delta x = 54t \rightarrow \Delta x = 54(3.0)$

$\boxed{\Delta x = 160\text{ m}}$

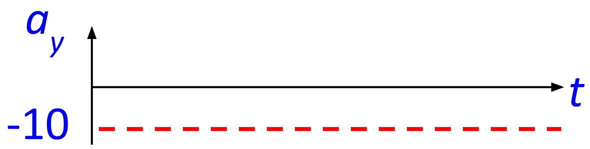
$v_x = 54$ $v_y = 15 - 10t$

$\Delta x = 54t$ $\Delta y = 15t - 5t^2$

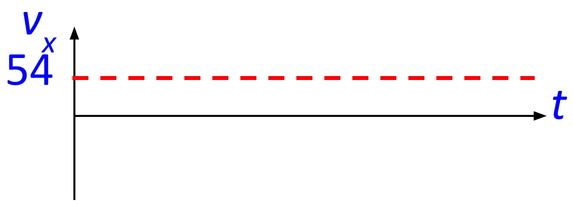
(e) Sketch the following graphs:

a vs. t, v_x vs. t, v_y vs. t:

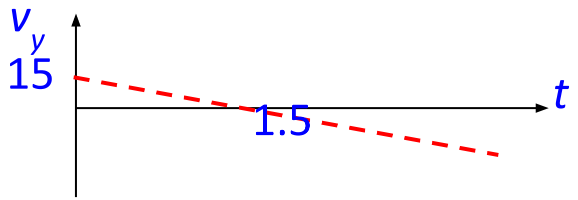
The only acceleration is g in the y-direction



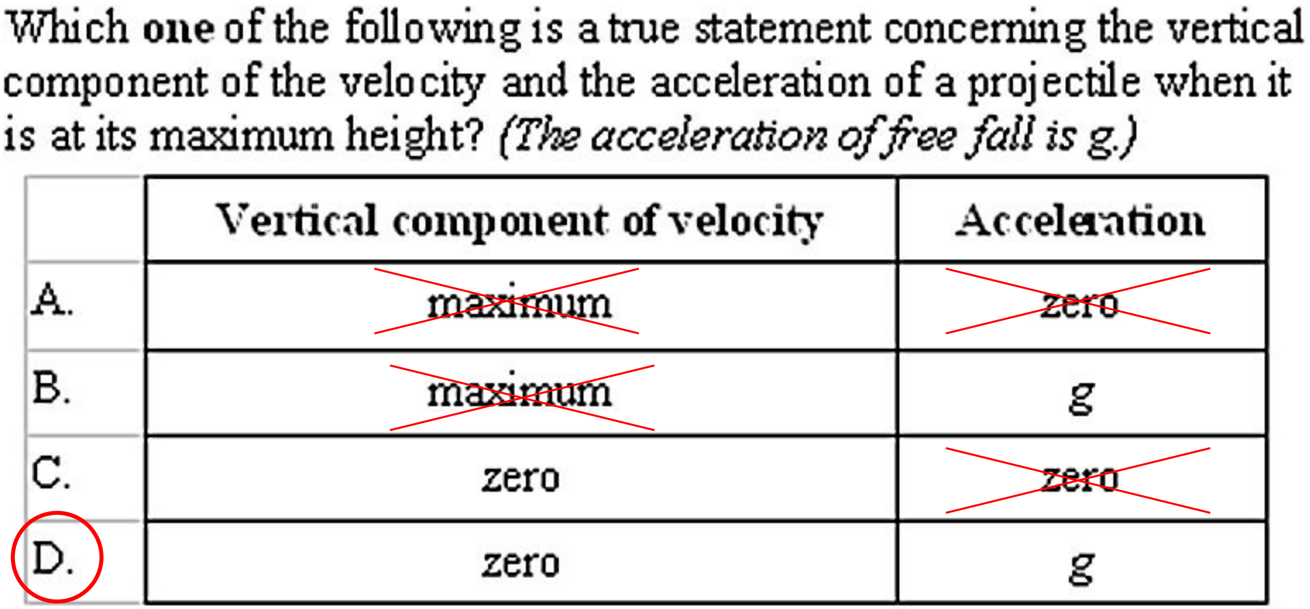
$v_x = 54\text{ ms}^{-1}$, a constant over time.



$v_y = 15 - 10t$
linear with a negative gradient and it crosses the time axis at 1.5 s.



Analysing projectile motion

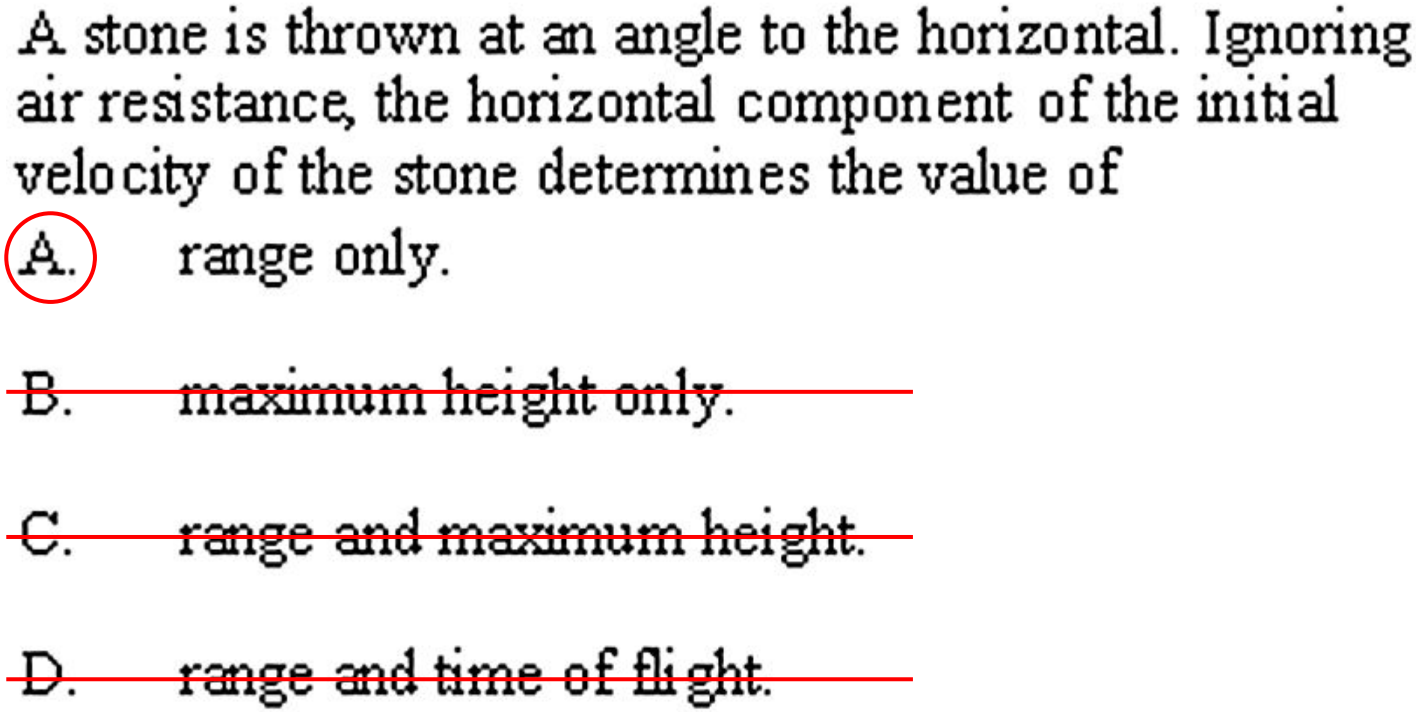


Which one of the following is a true statement concerning the vertical component of the velocity and the acceleration of a projectile when it is at its maximum height? *(The acceleration of free fall is g.)*

	Vertical component of velocity	Acceleration
A.	~~maximum~~	~~zero~~
B.	~~maximum~~	*g*
C.	zero	~~zero~~
[✓ D.]	zero	*g*

- The acceleration is ALWAYS *g* for projectile motion-since it is caused by Earth and its field.
- At the maximum height the projectile switches from upward to downward motion. $v_y = 0$ at the top.

Analysing projectile motion



A stone is thrown at an angle to the horizontal. Ignoring air resistance, the horizontal component of the initial velocity of the stone determines the value of

[✓ A.] range only.

~~B. maximum height only.~~

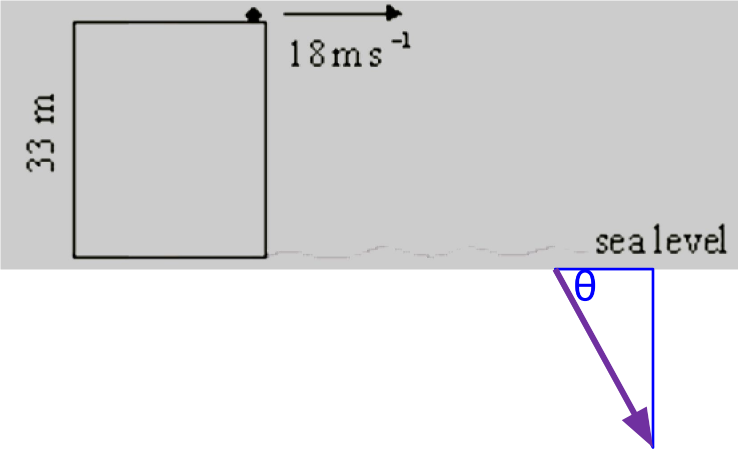
~~C. range and maximum height.~~

~~D. range and time of flight.~~

- The flight time is depends on vertical motion (u_y).
- The maximum height is on vertical motion (u_y).

Analysing projectile motion

A stone is thrown horizontally from the top of a vertical cliff of height 33 m as shown.



The initial horizontal velocity of the stone is 18 ms^{-1} and air resistance may be assumed to be negligible.

(a) State values for the horizontal and for the vertical acceleration of the stone.

Horizontal acceleration: $a_x = 0$.

Vertical acceleration: $a_y = -10$ ms^{-2}.

(b) Determine the time taken for the tone to reach sea level.

- Fall time determined by the height: $\Delta y = u_y t - 5t^2 \rightarrow -33 = 0t - 5t^2$

$\boxed{t = 2.6 \text{ s}}$

(c) Calculate the distance of the stone from the base of the cliff when it reaches sea level.

- find Δx at $t = 2.6$ s: $\Delta x = u_x t \rightarrow \Delta x = 18(2.6)$

$\boxed{\Delta x = 47 \text{ m.}}$

$v_y = -10(2.6) = -26.$

·tan θ = 26/18

$\theta = \tan^{-1}(26/18) = 55º.$

(d) Calculate the angle that the velocity makes with the surface of the sea.

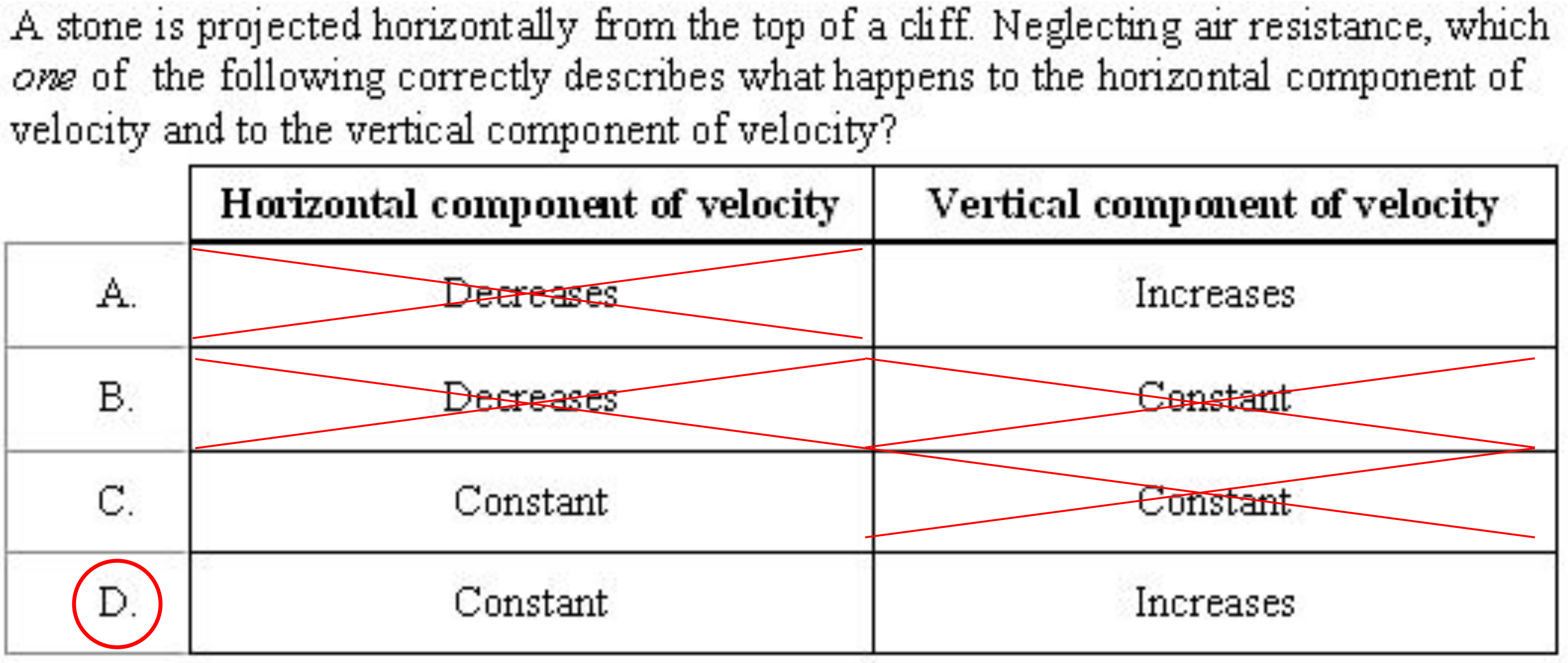
$v_x = u_x = 18$ms^{-1}

$v_y = u_y - 10t = -26$ms^{-1}

tan θ = 26/18

$\boxed{\theta = 55^0}$

Analysing projectile motion



A stone is projected horizontally from the top of a cliff. Neglecting air resistance, which *one* of the following correctly describes what happens to the horizontal component of velocity and to the vertical component of velocity?

	Horizontal component of velocity	**Vertical component of velocity**
A.	~~Decreases~~	Increases
B.	~~Decreases~~	~~Constant~~
C.	Constant	~~Constant~~
[✓ D.]	Constant	Increases

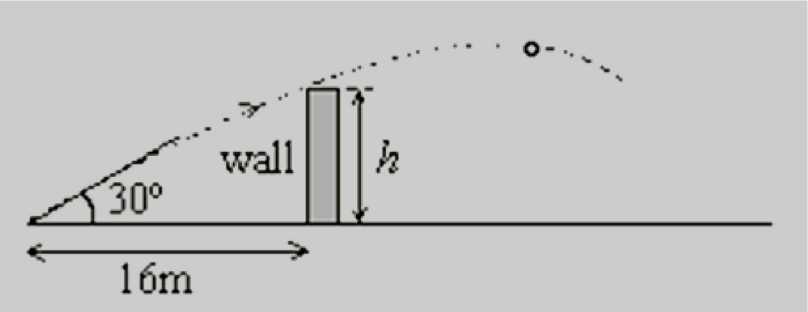
- The horizontal component of velocity is $v_x = u_x$ which is CONSTANT.
- The vertical component of velocity is $v_y = u_y - 10t$, which is INCREASING (negatively).

Mathematically strictly speaking, it is decreasing, only magnitude is increasing.

Analysing projectile motion

A ball is projected from ground level with a speed of 28 ms^{-1} at an angle of 30^{0} to the horizontal as shown.

There is a wall of height *h* at a distance of 16 m from the point of projection of the ball.
Air resistance is negligible.



(a) Calculate the initial magnitudes of

(i) The horizontal velocity of the ball

$$\boxed{u_x = u\cos 30^0 = 24ms^{-1}}$$

(ii) The vertical velocity of the ball

$$\boxed{u_y = u\sin 30^0 = 14ms^{-1}}$$

(b) The ball just passes over the wall. Determine maximum height of the wall.

- the time to the wall is found from Δx

$\Delta x = u_x t \quad \rightarrow \quad t = 16 / 24 = 0.67s$

$\Delta y = u_y t - 5t^2 \qquad \Delta y = 14(0.67) - 5(0.67)^2$

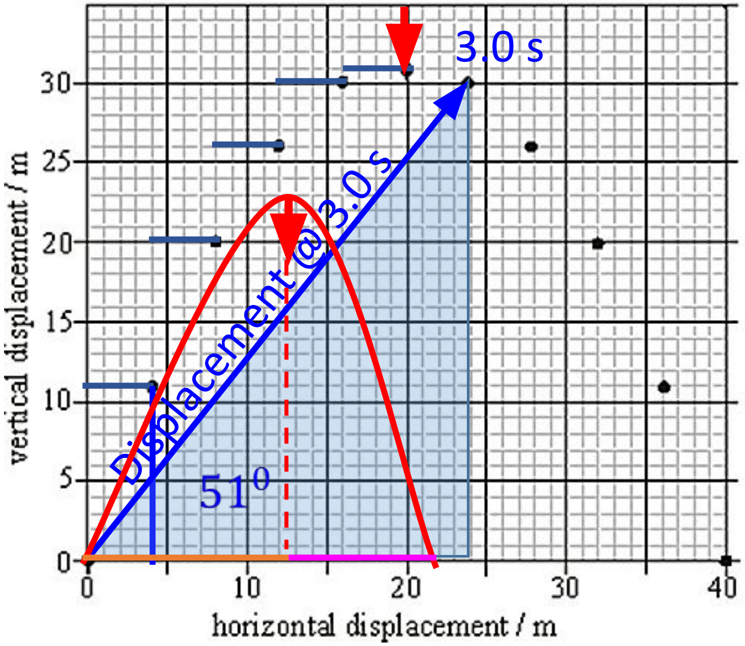
$$\boxed{\Delta y = 7.1 \text{ m}}$$

Analysing projectile motion

- New peak below and left.
- Pre-peak greater than post-peak.

A ball is kicked at an angle to the horizontal.
The diagram shows the position of the ball every 0.5 s.

The acceleration of free fall is g=10 ms^{-1} .
Air resistance may be neglected.



(a) Using the diagram determine, for the ball

(i) The horizontal component of the initial velocity

$$\Delta x = u_x t \rightarrow u_x = \Delta x/t = 4m/0.50s$$

$$\boxed{u_x = 8\text{ms}^{-1}}$$

(ii) The vertical component of the initial velocity

$$\Delta y = u_y t - 5t^2 \rightarrow u_y = \Delta y/t + 5t$$

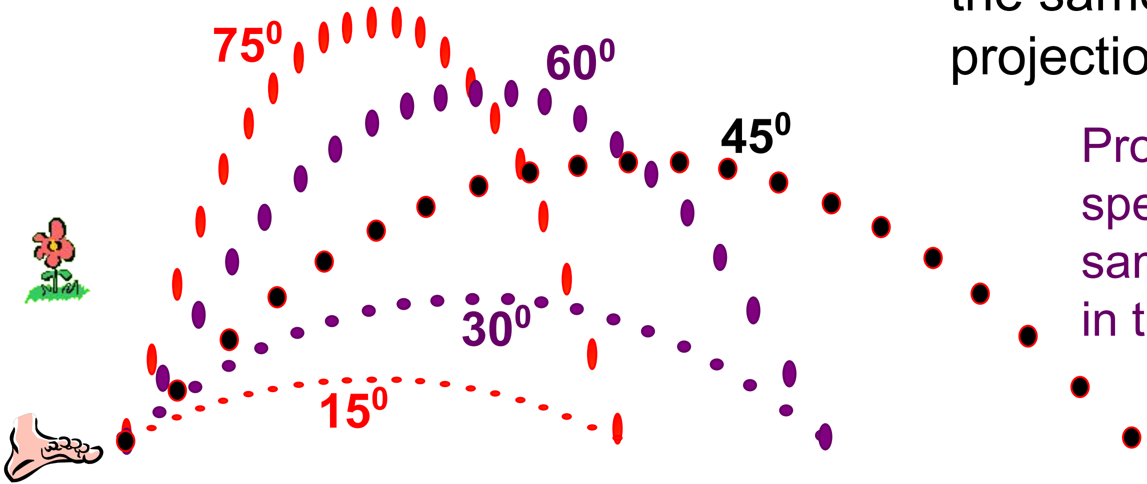
$$\boxed{u_y = 24.5\text{ms}^{-1} \approx 25\text{ms}^{-1}}$$

(iii) the displacement after 3.0 s. $\vec{D} = 38\ m\ @\ 51^0$

$$mag.of\ displacement = \sqrt{24^2 + 30^2} = 38m \qquad \theta = arc\tan\frac{30}{24} = 51^0$$

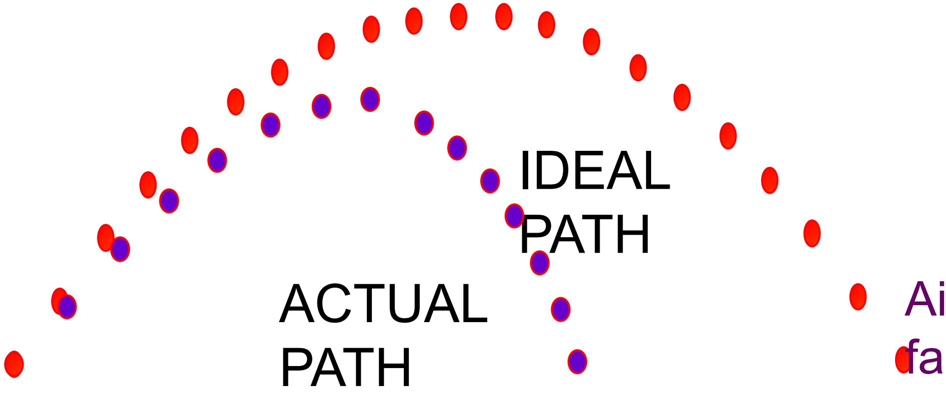
(b) On the diagram above draw a line to indicate a possible path for the ball if air resistance were not negligible.

the same range is obtained for two projection angles that add up to 90^0



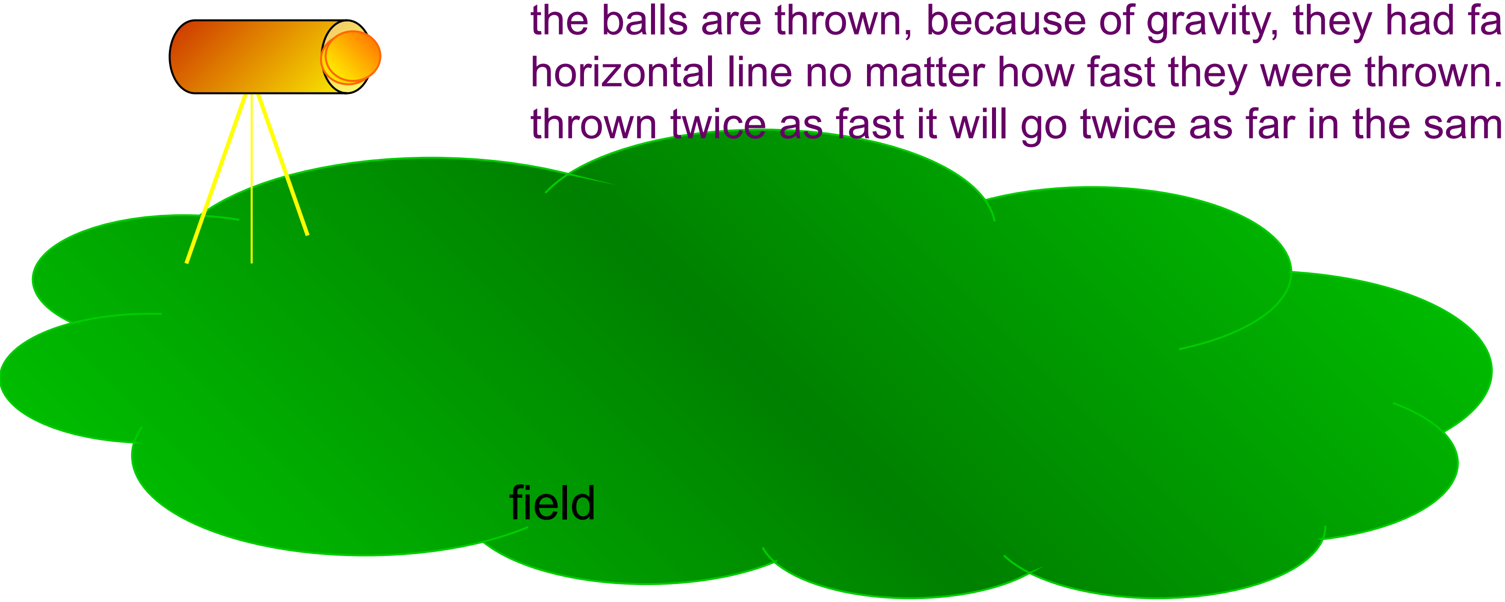
Projectile thrown with the same speed at 30^0 and 60^0 will have the same range. The one at 30^0 remains in the air for a shorter time.

Till now we neglected we either neglected or considered case in which air resistance was negligible. When the air resistance is significant, the range of projectile getting much smaller and the path is not true parabola. It is beyond the scope of this course to mathematically attack this problem.



Air resistance is particularly significant for fast-moving objects. A batted baseball travels only about 60% as far in air as it would in a vacuum. The path is no longer parabola.

The balls are thrown horizontally. If there was no gravity the balls would continue horizontally for ever at the same speed. But one second after the balls are thrown, because of gravity, they had fallen 5 m below horizontal line no matter how fast they were thrown. If the second ball is thrown twice as fast it will go twice as far in the same time..



If the ball were thrown three times as fast, it will go three times as far in the same time. Ten times as fast, ten times further.

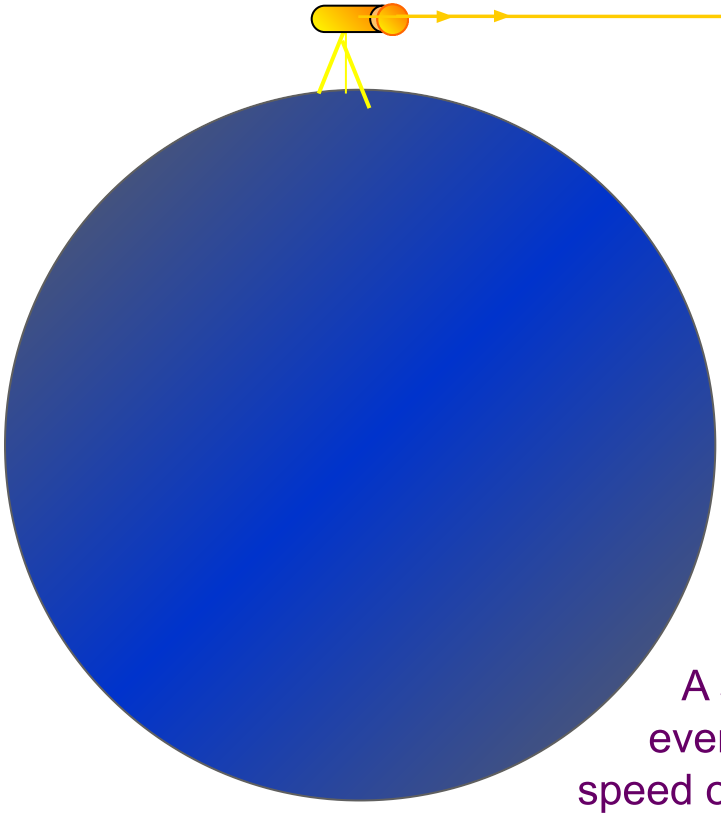
What if the ball were thrown so fast that the curvature of the earth came into play???

The ball follows a curved down path (parabola). But the Earth is curved too.

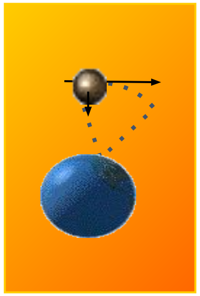
What if the ball were thrown so fast that the curved path matched the Earth’s curve?

If there were no air resistance (no slowing down and eventually hitting the Earth) the ball would orbit Earth.

An Earth satellite (space station, communication satellite, scientific satellites,…) is simply projectile traveling fast enough to fall around the Earth rather than into it.



Satellites in circular orbit, such as the moon or space station, fall beneath the paths they would follow if there were no gravity – straight line.



During each second the moon travels about one km . In this distance it deviates about one millimeter from a straight line due to the earth’s gravitational pull (dotted line). The moon continually falls toward the earth, as do the planets around the sun.

A satellite launched with speeds less than 8 km/s would eventually fall to the Earth. A satellite launched with a speed of 8 km/s would orbit the Earth in a circular path. Launched with a greater speed satellite would orbit the Earth in an elliptical path. If launched with too great of a speed, a satellite/projectile will escape Earth's gravitational influences and continue in motion without actually orbiting the Earth. Such a projectile will continue in motion until influenced by the gravitational influences of other celestial bodies.

At 8 km/s atmospheric friction would melt a piece of iron (falling stars). Therefore satellites are launched to altitudes above 150 km. Don’t even try to think they are free of Earth’s gravity. The force of gravity at that altitude is almost as strong as it is at the surface. The only reason we put them there is that they are beyond Earth's atmosphere , not beyond Earth’s gravity.