Uncertainties and error in experimental measurement

ERRORS – RANDOM AND SYSTEMATIC (PRECISION AND ACCURACY)

An experimental error just means that there is a difference between the recorded value and the 'perfect' or 'correct' value. Errors can be categorised as **random** or **systematic.**

Repeating readings does not reduce systematic errors.

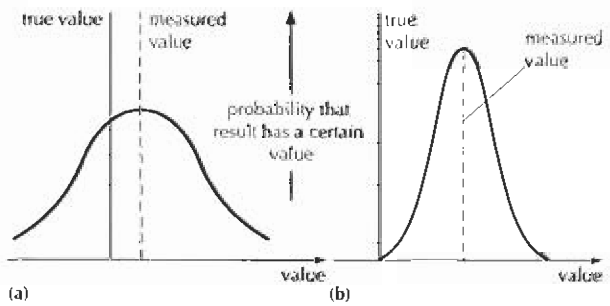
Sources of random errors include

- The readability of the instrument
- The observer being less than perfect
- The effects of a change in the surroundings.

Sources of systematic errors include

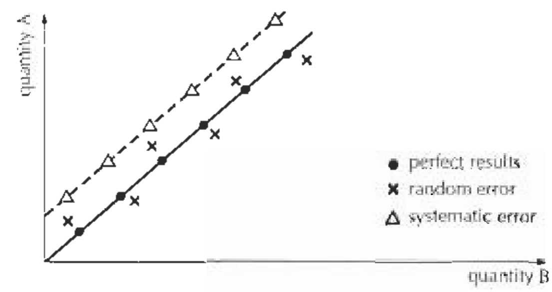
- An instrument **with zero error**. To correct for zero error the value should be subtracted from every reading.
- An instrument being wrongly **calibrated**
- The observer being less than perfect in the same way every measurement

An **accurate** experiment is one that has a small systematic error, whereas a **precise** experiment is one that has a small random error.



Two examples illustrating the nature of experimental results:
(a) an accurate experiment of low precision
(b) a less accurate but more precise experiment.

Systematic and random errors can often be recognised from a graph of the results.

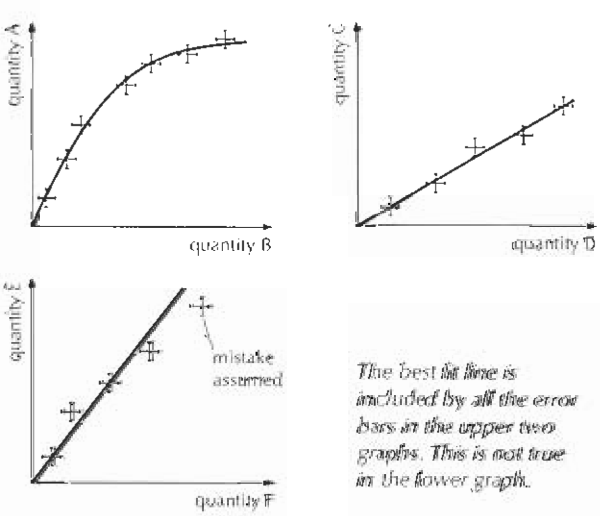


Perfect results, random and systematic errors of two proportional quantities.

GRAPHICAL REPRESENTATION OF UNCERTAINTY

In many situations the best method of presenting and analysing data is to use a graph. If this is the case, a neat way of representing the uncertainties is to use **error bars**. The graphs below explains their use.

Since the error bar represents the uncertainty range, the 'best-fit' line of the graph should pass through ALL of the rectangles created by the error bars.



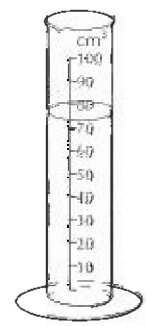
The best fit line is included by all the error bars in the upper two graphs. This is not true in the lower graph.

ESTIMATING THE UNCERTAINTY RANGE

An **uncertainty range** applies to any experimental value. The idea is that, instead of just giving one value that implies perfection, we give the likely range for the measurement.

1. Estimating from first principles

All measurement involves a readability error. If we use a measuring cylinder to find the volume of a liquid, we might think that the best estimate is 73 cm^3, but we know that it is not exactly this value (73.000000000 cm^3).



Uncertainty range is ± 5 cm^3.
We say volume = 73 ± 5 cm^3.

Normally the uncertainty range due to readability is estimated as below.

Device	Example	Uncertainty
Analogue scale	Rulers, meters with moving pointers	± (half the smallest scale division)
Digital scale	Top-pan balances, digital meters	± (the smallest scale division)

2. Estimating uncertainty range from several repeated measurements

If the time taken for a trolley to go down a slope is measured five times, the readings in seconds might be 2.01, 1.82, 1.97, 2.16 and 1.94. The average of these five readings is 1.98 s. The deviation of the largest and smallest readings can be calculated (2.16 – 1.98 = 0.18; 1.98 – 1.82 = 0.16). The largest value is taken as the uncertainty range. In this example the time is 1.98 s ± 0.18 s

SIGNIFICANT DIGITS

Any experimental measurement should be quoted with its uncertainty. This indicates the possible range of values for the quantity being measured. At the same time, the number of **significant digits** used will act as a guide to the amount of uncertainty. For example, a measurement of mass which is quoted as 23.456 g implies an uncertainty of ± 0.001 g (it has five significant digits), whereas one of 23.5 g implies an uncertainty of ± 0.1 g (it has three significant digits).

A simple rule for calculations (multiplication or division) is to quote the answer to the same number of significant digits as the LEAST precise value that is used.

For a more complete analysis of how to deal with uncertainties in calculated results, see page 8.

Graphs

PLOTTING GRAPHS – AXES AND BEST FIT

The reason for plotting a graph in the first place is that it allows us to identify trends. To be precise, it allows us a visual way of representing the variation of one quantity with respect to another. When plotting graphs, you need to make sure that all of the following points have been remembered

- The graph should have a title. Sometimes they also need a key.
- The scales of the axes should be suitable – there should not, of course, be any sudden or uneven 'jumps' in the numbers.
- The inclusion of the origin has been thought about. Most graphs should have the origin included – it is rare for a graph to be improved by this being missed out. If in doubt include it. You can always draw a second graph without it if necessary.
- The final graph should, if possible, cover more than half the paper in either direction.
- The axes are labelled with both the quantity (e.g. current) AND the units (e.g. amps).
- The points are clear. Vertical and horizontal lines to make crosses are better than 45 degree crosses or dots.
- All the points have been plotted correctly.
- Error bars are included if appropriate.
- A best-fit trend line is added. This line NEVER just 'joins the dots' – it is there to show the overall trend.
- If the best-fit line is a curve, this has been drawn as a single smooth line.
- If the best-fit line is a straight line, this has been added WITH A RULER
- As a general rule, there should be roughly the same number of points above the line as below the line.
- Check that the points are randomly above and below the line. Sometimes people try to fit a best-fit straight line to points that should be represented by a gentle curve. If this was done then points below the line would be at the beginning of the curve and all the points above the line would be at the end, or vice versa.
- Any points that do not agree with the best-fit line have been identified.

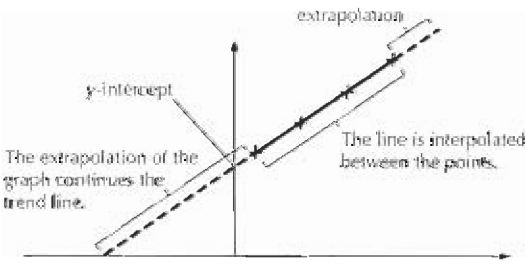
MEASURING INTERCEPT, GRADIENT AND AREA UNDER THE GRAPH

Graphs can be used to analyse the data. This is particularly easy for straight-line graphs, though many of the same principles can be used for curves as well. Three things are particularly useful: the **intercept**, the **gradient** and the **area under the graph**.

1. Intercept

In general, a graph can intercept (cut) either axis any number of times. A straight-line graph can only cut each axis once and often it is the ***y*-intercept** that has particular importance. (Sometimes the *y*-intercept is referred to as simply 'the intercept'.) If a graph has an intercept of zero it goes through the origin. **Proportional** – note that two quantities are proportional if the graph is a straight line THAT PASSES THROUGH THE ORIGIN.

Sometimes a graph has to be 'continued on' (outside the range of the readings) in order for the intercept to be found. This process is known as **extrapolation**. The process of assuming that the trend line applies between two points is known as **interpolation**.



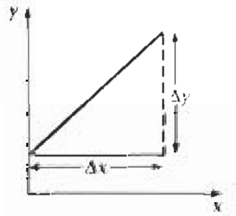
2. Gradient

The gradient of a straight-line graph is the increase in the *y*-axis value divided by the increase in the *x*-axis value.

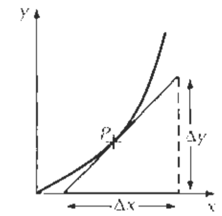
The following points should be remembered

- A straight-line graph has a constant gradient.
- The triangle used to calculate the gradient should be as large as possible.
- The gradient has units. They are the units on the *y*-axis divided by the units on the *x*-axis.
- Only if the *x*-axis is a measurement of time does the gradient represent the RATE at which the quantity on the *y*-axis increases.

The gradient of a curve at any particular point is the gradient of the tangent to the curve at that point.



gradient of straight line = $\frac{\Delta y}{\Delta x}$

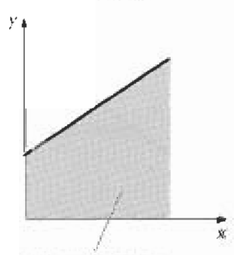


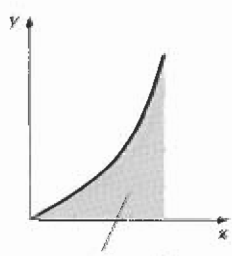
at point *P* on the curve, gradient = $\frac{\Delta y}{\Delta x}$

3. Area under a graph

The area under a straight-line graph is the product of multiplying the average quantity on the *y*-axis by the quantity on the *x*-axis. This does not always represent a useful physical quantity. When working out the area under the graph

- If the graph consists of straight-line sections, the area can be worked out by dividing the shape up into simple shapes.
- If the graph is a curve, the area can be calculated by 'counting the squares' and working out what one square represents.
- The units for the area under the graph are the units on the *y*-axis multiplied by the units on the *x*-axis.
- If the mathematical equation of the line is known, the area of the graph can be calculated using a process called **integration**.





Graphical analysis and determination of relationships

EQUATION OF A STRAIGHT-LINE GRAPH

All straight-line graphs can be described using one general equation

$$y = mx + c$$

y and x are the two variables (to match with the y-axis and the x-axis).

m and c are both constants – they have one fixed value.

- c represents the intercept on the y-axis (the value y takes when $x = 0$)
- m is the gradient of the graph.

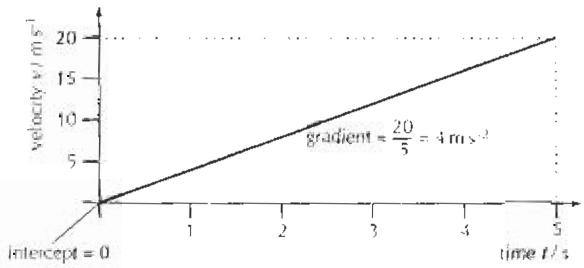
In some situations, a direct plot of the measured variable will give a straight line. In some other situations we have to choose carefully what to plot in order to get a straight line. In either case, once we have a straight line, we then use the gradient and the intercept to calculate other values.

For example, a simple experiment might measure the velocity of a trolley as it rolls down a slope. The equation that describes the motion is $v = u + at$ where u is the initial velocity of the object. In this situation v and t are our variables, a and u are the constants.

You should be able to see that the physics equation has exactly the same form as the mathematical equation. The order has been changed below so as to emphasis the link.

$$v = u + at$$
$$y = c + mx$$

By comparing these two equations, you should be able to see that if we plot the velocity on the y-axis and the time on the x-axis we are going to get a straight-line graph.



The comparison also works for the constants.

- c (the y-intercept) must be equal to the initial velocity u
- m (the gradient) must be equal to the acceleration a

In this example the graph tells us that the trolley must have started from rest (intercept zero) and it had a constant acceleration of 4.0 m s^{-2}.

CHOOSING WHAT TO PLOT TO GET A STRAIGHT LINE

With a little rearrangement we can often end up with the physics equation in the same form as the mathematical equation of a straight line. Important points include

- Identify which symbols represent variables and which symbols represent constants.
- The symbols that correspond to x and y must be variables and the symbols that correspond to m and c must be constants.
- If you take a variable reading and square it (or cube, square root, reciprocal etc) – the result is still a variable and you could choose to plot this on one of the axes.
- You can plot any mathematical combination of your original readings on one axis – this is still a variable.
- Sometimes the physical quantities involved use the symbols m (e.g. mass) or c (e.g. speed of light). Be careful not to confuse these with the symbols for gradient or intercept.

Example 1

The gravitational force F that acts on an object at a distance r away from the centre of a planet is given by the equation

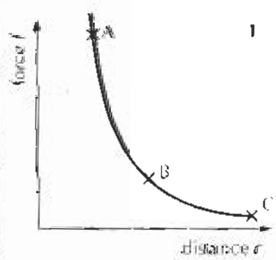
$F = \frac{GMm}{r^2}$ where M is the mass of the planet and the m is mass of the object.

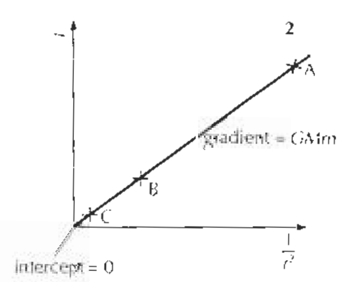
If we plot force against distance we get a curve (graph 1).

We can restate the equation as $F = \frac{GMm}{r^2} + 0$

and if we plot F on the y-axis and $\frac{1}{r^2}$ on the x-axis

we will get a straight-line (graph 2).





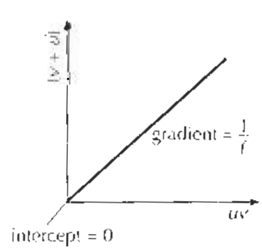
Example 2

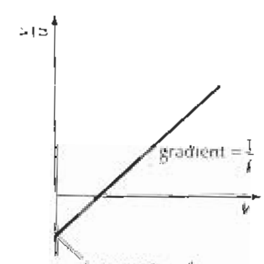
If an object is placed in front of a lens we get an image. The image distance v is related to the object distance u and the focal length of the lens f by the following equation.

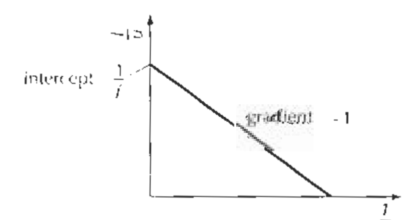
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

There are many possible ways to rearrange this in order to get it into straight–line form. You should check that all these are algebraically the same.

$$v + u = \frac{uv}{f} \quad \text{or} \quad \frac{v}{u} = \frac{v}{f} - 1 \quad \text{or} \quad \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$







Graphical analysis – logarithmic functions

LOGS – BASE TEN AND BASE e

Mathematically,

If $a = 10^{b}$

Then $\log(a) = b$

(to be absolutely precise $\log_{10}(a) = b$)

Most calculators have a 'log' button on them. But we don't **have** to use 10 as the base. We can use any number that we like. For example we could use 2.0, 563.2, 17.5, 42 or even 2.7182818284590452353602874714. For complex reasons this last number IS the most useful number to use! It is given the symbol e and logarithms to this base are called **natural logarithms**. The symbol for natural logarithms is ln (x). This is also on most calculators.

If $p = e^{q}$

Then $\ln(p) = q$

The powerful nature of logarithms means that we have the following rules

$\ln(c \times d) = \ln(c) + \ln(d)$

$\ln(c \div d) = \ln(c) - \ln(d)$

$\ln(c^{n}) = n \ln(c)$

$\ln\left(\frac{1}{c}\right) = -\ln(c)$

These rules have been expressed for natural logarithms, but they work for all logarithms whatever the base.

The point of logarithms is that they can be used to express some relationships (particularly power laws and exponentials) in straight line form. This means that we will be plotting graphs with logarithmic scales.

A normal scale increases by the same amount each time.

1 2 3 4 5 6 7 8 9 10 11

A logarithmic scale increases by the same ratio all the time.

10^0 10^1 10^2 10^3

1 10 100 1000

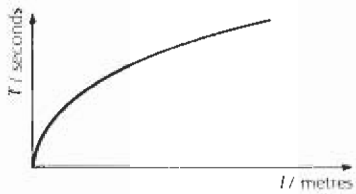
POWER LAWS AND LOGS (LOG-LOG)

When an experimental situation involves a power law it is often only possible to transform it into straight-line form by taking logs. For example, the time period of a simple pendulum, T, is related to its length, l, by the following equation.

$T = k\,l^{p}$

k and p are constants.

A plot of the variables will give a curve, but it is not clear from this curve what the values of k and p work out to be. On top of this, if we do not know what the value of p is, we can not calculate the values to plot a straight-line graph.



Time period versus length for a simple pendulum

The 'trick' is to take logs of both sides of the equation. The equations below have used natural logarithms, but would work for all logarithms whatever the base.

$\ln(T) = \ln(k\,l^{p})$

$\ln(T) = \ln(k) + \ln(l^{p})$

$\ln(T) = \ln(k) + p\ln(l)$

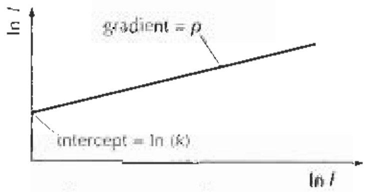
This is now in the same form as the equation for a straight-line

$y = c + mx$

Thus if we plot ln (T) on the y-axis and ln (l) on the x-axis we will get a straight-line graph.

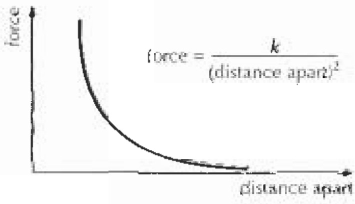
The gradient will be equal to p

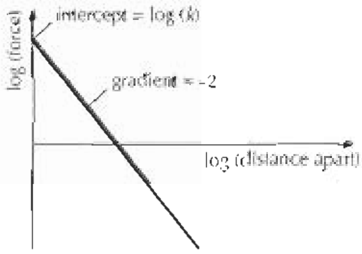
The intercept will be equal to $\ln(k)$ [so $k = e^{(\text{intercept})}$]



Plot of ln (time period) versus ln (length) gives a straight-line graph.

Both the gravity force and the electrostatic force are inverse-square relationships. This means that the force $\propto$ (distance apart)$^{-2}$. The same technique can be used to generate a straight-line graph.





Inverse square relationship – direct plot and log-log plot

EXPONENTIALS AND LOGS (LOG-LINEAR)

Natural logarithms are very important because many natural processes are exponential. Radioactive decay is an important example. In this case, once again the taking of logarithms allows the equation to be compared with the equation for a straight line.

For example, the count rate R at any given time t is given by the equation

$R = R_0\,e^{-\lambda t}$

R_0 and λ are constants.

If we take logs, we get

$\ln(R) = \ln(R_0\,e^{-\lambda t})$

$\ln(R) = \ln(R_0) + \ln(e^{-\lambda t})$

$\ln(R) = \ln(R_0) - \lambda t \ln(e)$

$\ln(R) = \ln(R_0) - \lambda t$ [$\ln(e) = 1$]

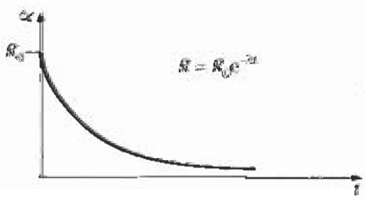
This can be compared with the equation for a straight-line graph

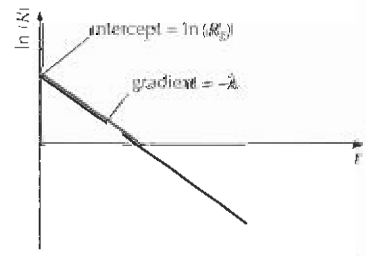
$y = c + mx$

Thus if we plot ln (R) on the y-axis and t on the x-axis, we will get a straight line.

Gradient = $-\lambda$

Intercept = $\ln(R_0)$





Exponential decrease – direct plot and log-linear plot

Uncertainties in calculated results

MATHEMATICAL REPRESENTATION OF UNCERTAINTIES

For example if the mass of a block was measured as 10 ± 1 g and the volume was measured as 5.0 ±0.2 cm^3, then the full calculations for the density would be as follows.

Best value for density = $\frac{\text{mass}}{\text{volume}} = \frac{10}{5} = 2.0 \text{ g cm}^{-3}$

The largest possible value of density = $\frac{11}{4.8} = 2.292 \text{ g cm}^{-3}$

The smallest possible value of density = $\frac{9}{5.2} = 1.731 \text{ g cm}^{-3}$

Rounding these values gives density = $2.0 \pm 0.3 \text{ g cm}^{-3}$

We can express this uncertainty in one of three ways – using **absolute, fractional** or **percentage uncertainties**.

If a quantity p is measured then the absolute uncertainty would be expressed as $\pm\Delta p$.

Then the fractional uncertainty is

$$\frac{\pm\Delta p}{p},$$

which makes the percentage uncertainty

$$\frac{\pm\Delta p}{p} \times 100\%.$$

In the example above, the fractional uncertainty is ±0.15 or ±15%.

Thus equivalent ways of expressing this error are

density = $2.0 \pm 0.3 \text{ g cm}^{-3}$

OR density = $2.0 \text{ g cm}^{-3} \pm 15\%$

Working out the uncertainty range is very time consuming. There are some mathematical 'short-cuts' that can be used. These are introduced in the boxes below.

MULTIPLICATION, DIVISION OR POWERS

Whenever two or more quantities are multiplied or divided and they each have uncertainties, the overall uncertainty is approximately equal to the **addition** of the **percentage** (fractional) **uncertainties**.

Using the same numbers from above,

$\Delta m = \pm 1 \text{ g}$

$$\frac{\Delta m}{m} = \pm\left(\frac{1 \text{ g}}{10 \text{ g}}\right) = \pm 0.1 = \pm 10\%$$

$\Delta v = \pm 0.2 \text{ cm}^3$

$$\frac{\Delta V}{V} = \pm\left(\frac{0.2 \text{ cm}^3}{5 \text{ cm}^3}\right) = \pm 0.04 = \pm 4\%$$

The total % uncertainty in the result = ± (10 + 4)%
= ± 14 %

14% of $2.0 \text{ g cm}^{-3} = 0.28 \text{ g cm}^{-3} \approx 0.3 \text{ g cm}^{-3}$

So density = $2.0 \pm 0.3 \text{ g cm}^{-3}$ as before.

In symbols, if $P = Q \times R$ or if $P = \frac{Q}{R}$

Then $\frac{\Delta P}{P} = \frac{\Delta Q}{Q} + \frac{\Delta R}{R}$ [note this is ALWAYS added]

Power relationships are just a special case of this law.

If $P = R^n$

Then $\frac{\Delta P}{P} = n\left(\frac{\Delta R}{R}\right)$

For example if a cube is measured to be 4.0 ± 0.1 cm in length along each side, then

% Uncertainty in length = $\pm\left(\frac{0.1}{4.0}\right) = \pm 2.5\%$

Volume = $(\text{length})^3 = (4.0)^3 = 64 \text{ cm}^3$

% Uncertainty in [volume] = % uncertainty in [$(\text{length})^3$]
= 3×(% uncertainty in [length])
= 3×(± 2.5 %)
= ± 7.5 %

Absolute uncertainty = 7.5% of 64 cm^3
= 4.8 cm^3 ≈ 5 cm^3

Thus volume of cube = 64 ± 5 cm^3

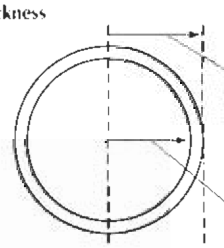
OTHER MATHEMATICAL OPERATIONS

If the calculation involves mathematical operations other than multiplication, division or raising to a power, then one has to find the highest and lowest possible values.

Addition or subtraction

Whenever two or more quantities are added or subtracted and they each have uncertainties, the overall uncertainty is equal to the **addition** of the **absolute** uncertainties.

uncertainty of thickness in a pipe wall



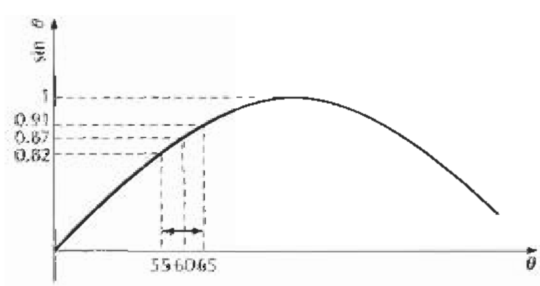
thickness of pipe wall = 6.1 – 5.3 cm
= 0.8 cm

uncertainty in thickness = ±(0.1 + 0.1) cm
= 0.2 cm
= ±25%

Other functions

There are no 'short-cuts' possible.

uncertainty of sin θ if $\theta = 60° \pm 5°$



if $\theta = 60° \pm 5°$

mean sin $\theta = 0.87$

max. sin $\theta = 0.91$

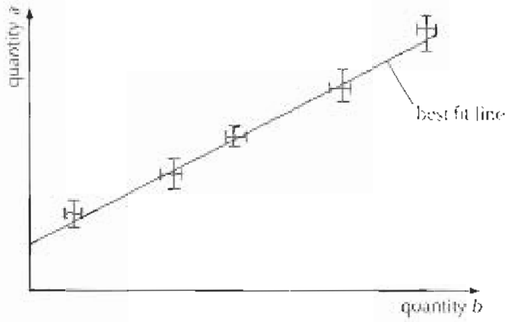
min. sin $\theta = 0.82$

∴ sin $\theta = 0.87 \pm 0.05$ (worst value used)

Uncertainties in graphs

ERROR BARS

Plotting a graph allows one to visualise all the readings at one time. Ideally all of the points should be plotted with their error bars. In principle, the size of the error bar could well be different for every single point and so they should be individually worked out.

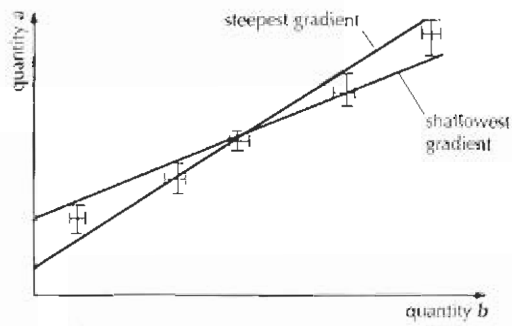


In practice, it would often take too much time to add all the correct error bars, so some (or all) of the following short cuts could be considered.

- Rather than working out error bars for each point – use the worst value and assume that all of the other error bars are the same.
- Only plot the error bar for the 'worst' point i.e. the point that is furthest from the line of best fit. If the line of best fit is within the limits of this error bar, then it will probably be within the limits of all the error bars.
- Only plot the error bars for the first and the last points. These are often the most important points when considering the uncertainty ranges calculated for the gradient or the intercept (see below).
- Only include the error bars for the axis that has the worst uncertainty.

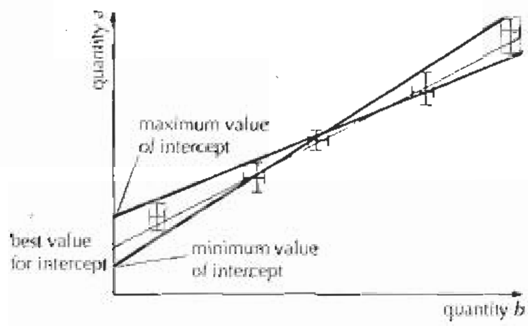
UNCERTAINTY IN SLOPES

If the gradient of the graph has been used to calculate a quantity, then the uncertainties of the points will give rise to an uncertainty in the gradient. Using the steepest and the shallowest lines possible (i.e. the lines that are still consistent with the error bars) the uncertainty range for the gradient is obtained. This process is represented below.



UNCERTAINTY IN INTERCEPTS

If the intercept of the graph has been used to calculate a quantity, then the uncertainties of the points will give rise to an uncertainty in the intercept. Using the steepest and the shallowest lines possible (i.e. the lines that are still consistent with the error bars) we can obtain the uncertainty in the result. This process is represented below.



IB QUESTIONS – PHYSICS AND PHYSICAL MEASUREMENT

1 Which one of the following quantities is a scalar?

A Weight
B Distance
C Velocity
D Momentum

2 Which one of the following is a fundamental unit in the International System of units (S.I.)?

A newton
B ampere
C joule
D pascal

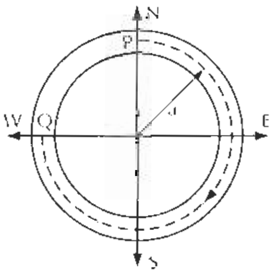
3 Gravitational field strength may be specified in N kg^{-1}. Units of N kg^{-1} are equivalent to

A m s^{-1}
B m s^{-2}
C kg m s^{-1}
D kg m s^{-2}

4 Which one of the following is a scalar quantity?

A Electric field
B Acceleration
C Power
D Momentum

5 A motor car travels on a circular track of radius, *a*, as shown in the figure. When the car has travelled from P to Q its displacement from P is



A $a\sqrt{2}$ southwest
B $a\sqrt{2}$ northeast
C $\frac{3\pi a}{2}$ southwest
D $\frac{3\pi a}{2}$ northeast

6 The frequency of oscillation *f* of a mass *m* suspended from a vertical spring is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where *k* is the spring constant.

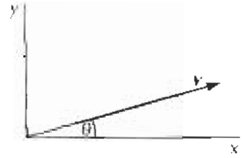
Which **one** of the following plots would produce a straight-line graph?

A f against m
B f^2 against $\frac{1}{m}$
C f against $\sqrt{m}$
D $\frac{1}{f}$ against m

7 Repeated measurements of a quantity can reduce the effects of

A both random and systematic errors
B only random errors
C only systematic errors
D neither random nor systematic errors

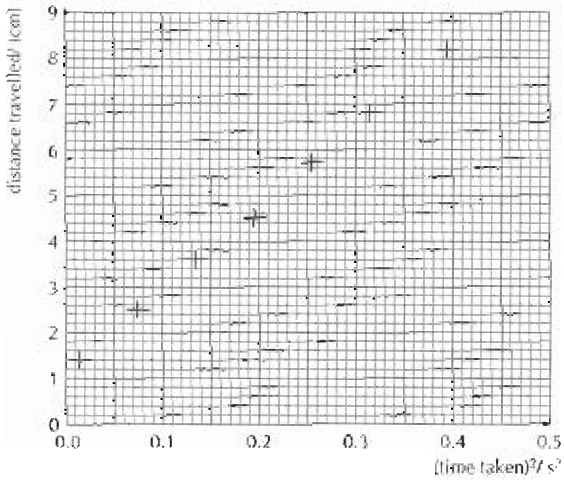
8 A vector **v** makes an angle θ with the x axis as shown.



As the angle θ increases from 0° to 90°, how do the x and y components of **v** vary?

	x component	y component
A	Increases	Increases
B	Increases	Decreases
C	Decreases	Increases
D	Decreases	Decreases

9 An object is rolled from rest down an inclined plane. The distance travelled by the object was measured at seven different times. A graph was then constructed of the distance travelled against the (time taken)^2 as shown below.



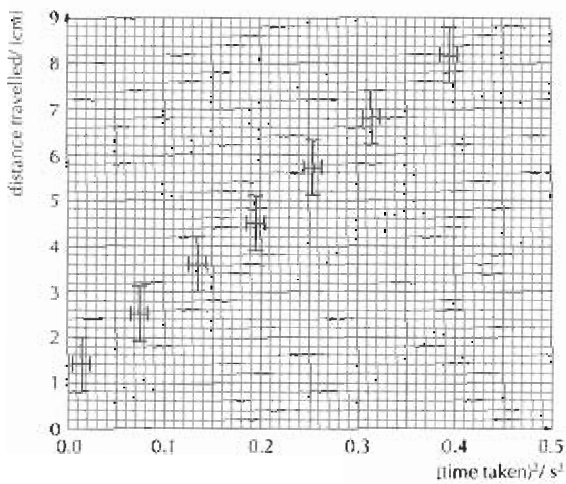
(a) **(i)** What quantity is given by the gradient of such a graph? [2]

(ii) Explain why the graph suggests that the collected data is valid but includes **a systematic error**. [2]

(iii) Do these results suggest that distance is proportional to (time taken)^2? Explain your answer. [2]

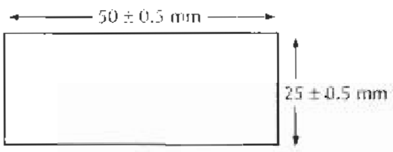
(iv) Making allowance for the systematic error, calculate the acceleration of the object. [2]

(b) The following graph shows that same data after the uncertainty ranges have been calculated and drawn as error bars.



Add two lines to show the range of the possible acceptable values for the gradient of the graph. [2]

10 The lengths of the sides of a rectangular plate are measured, and the diagram shows the measured values with their uncertainties.



Which one of the following would be the best estimate of the percentage uncertainty in the calculated area of the plate?

A ± 0.02% **C** ± 3%

B ± 1% **D** ± 5%

11 A stone is dropped down a well and hits the water 2.0 s after it is released. Using the equation $d = \frac{1}{2}g t^2$ and taking $g = 9.81$ m s^{-2}, a calculator yields a value for the depth d of the well as 19.62 m. If the time is measured to ±0.1 s then the best estimate of the absolute error in d is

A ±0.1 m **C** ±1.0 m

B ±0.2 m **D** ±2.0 m

12 In order to determine the density of a certain type of wood, the following measurements were made on a **cube** of the wood.

Mass = 493 g
Length of each side = 9.3 cm

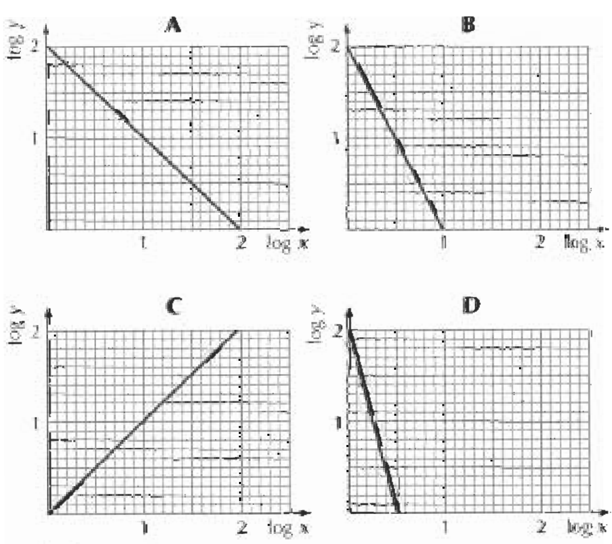
The percentage uncertainty in the measurement of mass is ±0.5% and the percentage uncertainty in the measurement of length is ±1.0%.

The best estimate for the uncertainty in the density is

A ±0.5% **C** ±3.0%

B ±1.5% **D** ±3.5%

13 The graphs A to D below are plots of log y against log x in arbitrary units.

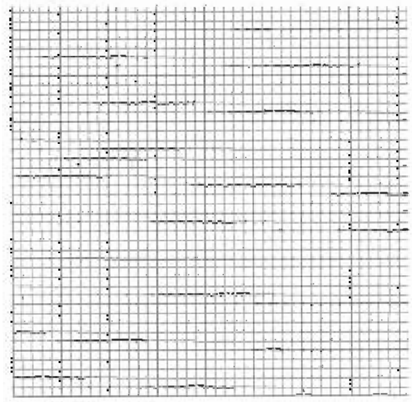


Which one of the graphs best represents the variation of y, the electrostatic potential due to a positive point charge, with x, the distance from the point charge?

14 This question is about finding the relationship between the forces between magnets and their separations.

In an experiment, two magnets were placed with their North-seeking poles facing one another. The force of repulsion, f, and the separation of the magnets, d, were measured and the results are shown in the table below.

Separation d/m	Force of repulsion f/N
0.04	4.00
0.05	1.98
0.07	0.74
0.09	0.32



(a) Plot a graph of log (force) against log (distance). [3]

(b) The law relating the force to the separation is of the form

$f = kd^n$

(i) Use the graph to find the value of n. [2]
(ii) Calculate a value for k, giving its units. [3]

15 Astronauts wish to determine the gravitational acceleration on Planet X by dropping stones from an overhanging cliff. Using a steel tape measure they measure the height of the cliff as $s = 7.64$ m ± 0.01 m. They then drop three similar stones from the cliff, timing each fall using a hand-held electronic stopwatch which displays readings to one-hundredth of a second. The recorded times for three drops are 2.46 s, 2.31 s and 2.40 s.

(a) Explain why the time readings vary by more than a tenth of a second, although the stopwatch gives readings to one hundredth of a second. [1]

(b) Obtain the average time t to fall, and write it in the form (value ± uncertainty), to the appropriate number of significant digits. [1]

(c) The astronauts then determine the gravitational acceleration a_g on the planet using the formula $a_g = \frac{2s}{t^2}$. Calculate a_g from the values of s and t, and determine the uncertainty in the calculated value. Express the result in the form a_g = (value ± uncertainty), to the appropriate number of significant digits. [3]