Polarization

This chapter introduces polarization, a property of transverse waves. A wave is polarized if the displacement of the wave always lies in the same plane. This chapter discusses how a wave can be polarized and introduces Malus's law for the intensity of light transmitted through a polarizer. We also discuss Brewster's law and close with a few applications of polarized light.

Objectives

By the end of this chapter you should be able to:

- explain the meaning of the term *polarization*;
- understand how *light can be polarized*;
- state and apply *Malus's law*;
- state and apply *Brewster's law*;
- understand the terms *optical activity* and *optically active substances*;
- outline some *applications of polarized light*, including the structure and operation of liquid crystal displays.

What is polarization?

Light (like all other transverse waves) has the important property of polarization. Before discussing the case of light, let us look at a simpler mechanical wave, a wave on a string. Figure 9.1 shows a string that is made to oscillate so that a transverse wave propagates along the string. In Figure 9.1(a) the string is always in the same vertical plane. In Figure 9.1(b) the string is always in a horizontal plane. The string waves here are said to be *plane polarized* because in each case the string is always in a fixed plane.

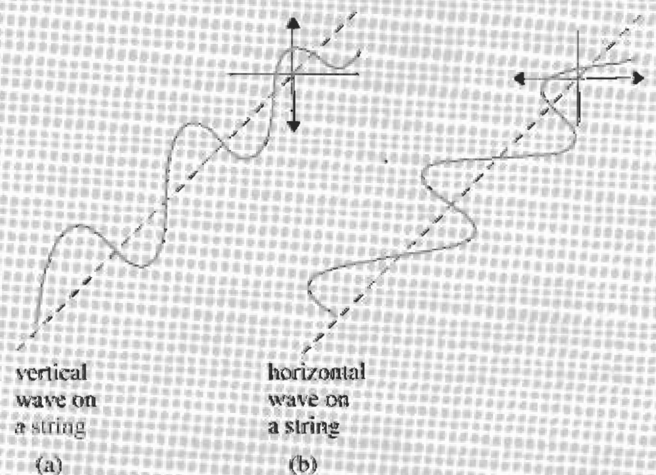

Figure 9.1 A string wave that is (a) vertically polarized and (b) horizontally polarized.

Now imagine a vertically polarized string wave. If an obstacle with a vertical slit is placed in the path of this wave (see Figure 9.2), the wave will simply go through the slit unimpeded. However, if the obstacle has a horizontal slit, the wave will be stopped, and no wave will be transmitted beyond the obstacle.

Like all other electromagnetic waves, light is a transverse wave in which an electric field and a magnetic field at right angles to each other propagate along a direction that is normal to both fields. For the discussion of the

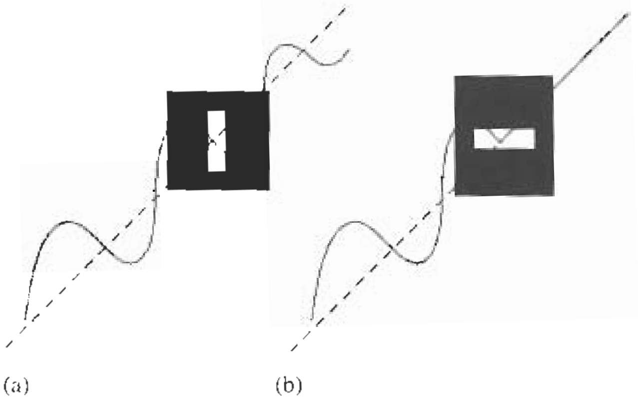

Figure 9.2 A vertical string wave passes through a vertical slit (a) . . . but not through a horizontal slit (b).

polarization of light, it is sufficient to concentrate only on the electric field in the electromagnetic wave and to ignore the magnetic field.

An electromagnetic wave is said to be **plane polarized** if the electric field always lies in the same plane, as the wave propagates. Thus in Figure 9.3(a) the wave is plane polarized, but in Figure 9.3(b) the wave is unpolarized. In both cases the wave is propagating along the direction into the plane of the page.

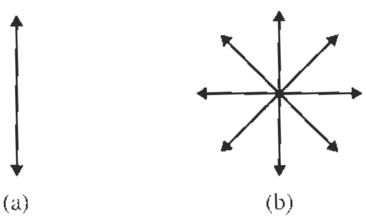

Figure 9.3 Electric field vectors of (a) polarized and (b) unpolarized light. Both waves are propagating into the plane of the page.

Most of the light around us, for example light from the sun or a light bulb, is unpolarized light. Unpolarized light can be polarized by letting it go through a **polarizer**. A polarizer is a sheet of material with a molecular structure that only allows a specific orientation of the electric field to go through (see Figure 9.4). The most common polarizer is a plastic called Polaroid invented by Edwin Land, a 19-year-old undergraduate at Harvard, in 1928. Thus a sheet of Polaroid with a vertical transmission axis

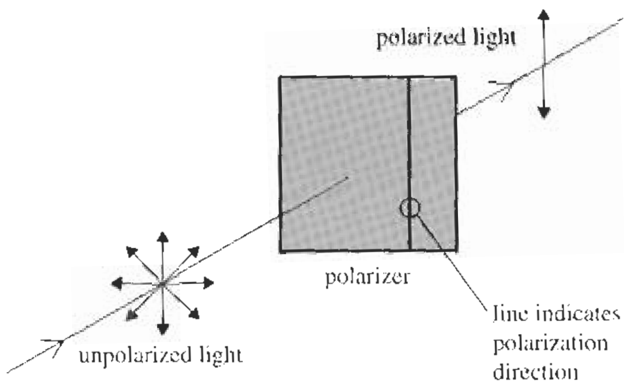

Figure 9.4 This polarizer only allows components of electric fields parallel to the vertical transmission axis to go through. Vertically polarized light is transmitted through this polarizer.

(this means only vertical electric fields can go through) placed in the path of unpolarized light will transmit only vertically polarized light. In diagrams, the transmission axis of the polarizer is indicated with a line.

Malus's law

Thus, consider an electromagnetic wave whose electric field E_0 makes an angle θ with the transmission axis of a polarizer. We may resolve the electric field into a component along the transmission axis and a component at right angles to it. Only the component along the axis will go through (see Figure 9.5).

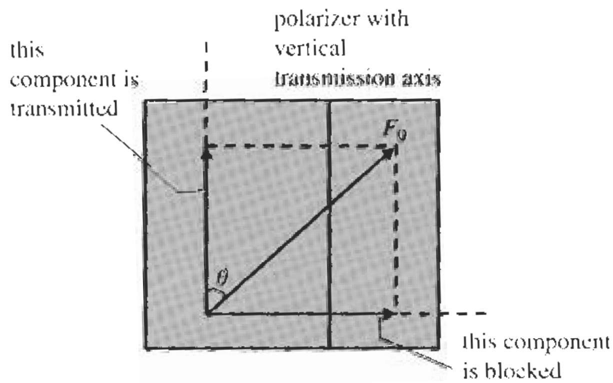

Figure 9.5 This polarizer has a vertical transmission axis. Therefore, only the component of the electric field along the vertical axis will be transmitted.

This component of the electric field along the transmission axis is

$$E = E_0 \cos\theta$$

The transmitted intensity I is proportional to the square of the electric field. So we have that

$$I = I_0 \cos^2\theta$$

where I_0 is the incident intensity. This is **Malus's law**, named after the Frenchman Etienne Malus, who studied this effect in 1808. The polarizer reduces the intensity of the transmitted light. We see that when the electric field is along the transmission axis ($\theta = 0$) then $I = I_0$, and when the electric field is at right angles to the transmission axis ($\theta = 90°$) then $I = 0$.

Example question

Q1

Vertically polarized light of intensity I_0 is incident on a polarizer that has its transmission axis at $\theta = 30°$ to the vertical. The transmitted light is then incident on a second polarizer whose axis is at $\theta = 60°$ to the vertical. Calculate the factor by which the transmitted intensity is reduced.

Answer

After passing through the first polarizer the intensity of light is

$$I = I_0 \cos^2\theta = I_0 \cos^2 30° = \frac{3I_0}{4}$$

The second polarizer has its transmission axis at $\theta = 30°$ to the first polarizer, and so the final transmitted light has intensity

$$I = \frac{3I_0}{4}\cos^2 30° = \frac{9I_0}{16}$$

The intensity is thus reduced by a factor of $\frac{9}{16}$.

Polarizers and analysers

A polarizer can be used to produce polarized light. It can also be used to determine if light is polarized. A polarizer used for this purpose is called an **analyser**. Unpolarized light passing through a polarizer (analyser) will have its intensity reduced by the same amount (by 50% in fact – see below) no matter what the orientation of the polarizer (analyser). Polarized light, on the other hand, will have its intensity reduced by an amount that depends on the orientation of the polarizer (analyser).

When unpolarized light is incident on a polarizer, the transmitted light will have its intensity reduced (since part of the light will be blocked by the polarizer). We can calculate the factor by which the intensity is reduced as follows. We think of the incident unpolarized light as having two electric fields, of equal magnitude, in directions along and normal to the transmission axis of the polarizer. The incident intensity is then proportional to $E^2 + E^2 = 2E^2$, where E is the magnitude of either the vertical or the horizontal electric field component. One of these components will be blocked, and so the transmitted intensity will be proportional to just E^2. Thus the intensity is reduced by a factor of 2 or 50% (Figure 9.6).

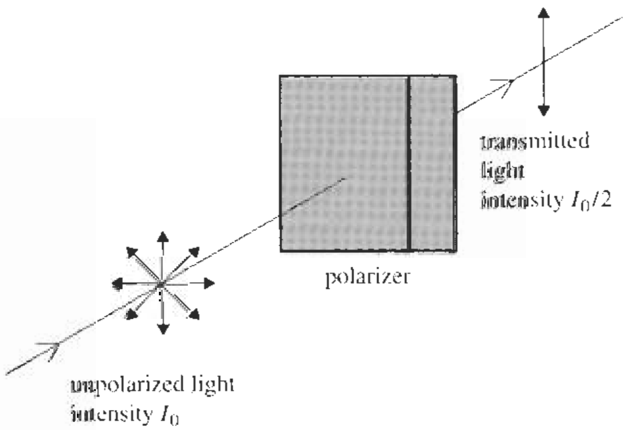

Figure 9.6 Unpolarized light has its intensity reduced by a factor of 2 after passing through a polarizer (analyser).

Supplementary material

For the more mathematically minded, the transmitted intensity will be, using Malus's law, $I = I_0 \cos^2\theta$. But each component of the incident unpolarized light will make a different angle θ with the transmission axis. Since we have a very large number of randomly chosen angles θ, we must find the average value of $\cos^2\theta$. This is just $\frac{1}{2}$, and so the transmitted intensity is half of the incident intensity.

When two polarizers are placed with their transmission axes at right angles to each other, no light emerges from the second polarizer (Figure 9.7).

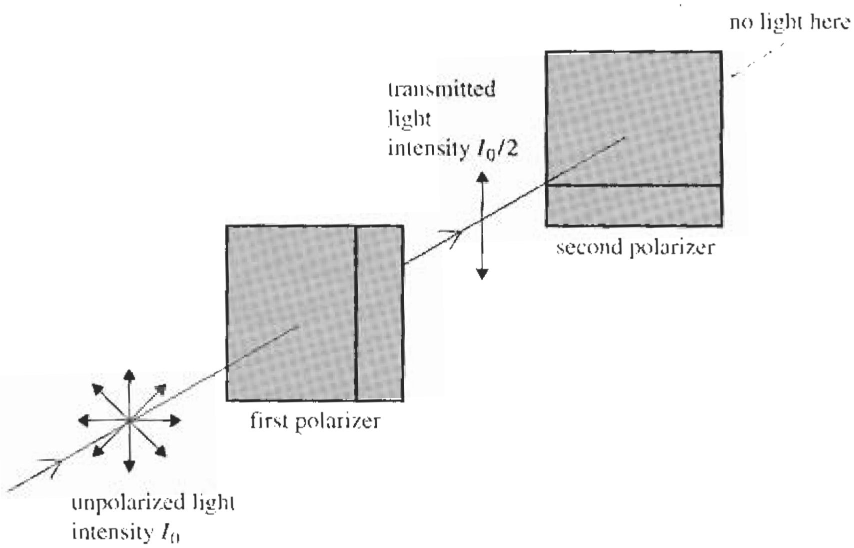

Figure 9.7 No light gets transmitted by an arrangement of two polarizers at right angles to each other.

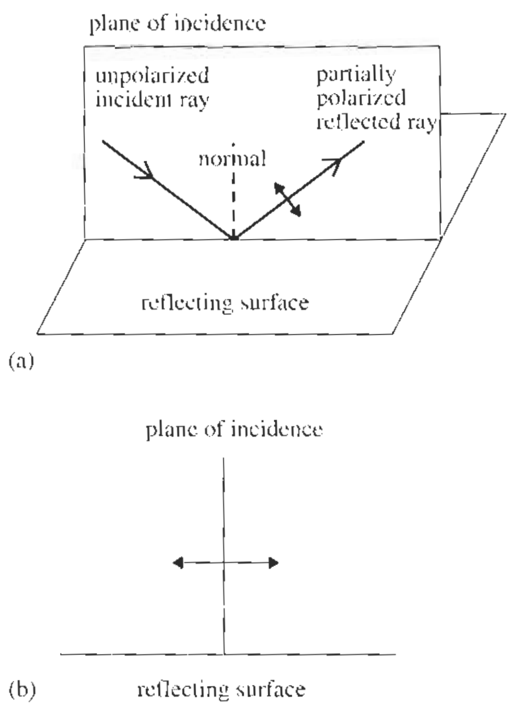

Figure 9.8 Partial polarization by reflection. (a) There is a small electric field component in the plane of incidence. (b) There is a larger electric field component in the plane parallel to the reflecting surface, as shown in this edge view.

Polarization by reflection

Polarized light can be obtained not only by passing light through a polarizer but also by reflection. When unpolarized light reflects off a non-metallic surface, the reflected ray is partially polarized (Figure 9.8). The 'glare' from reflections off the sea is partially polarized, and can be reduced by wearing Polaroid sunglasses (which have polarizing plastic lenses). The plane of polarization is parallel to the reflecting surface. Partially polarized light in this case means that the reflected light has various components of electric field of unequal magnitude. The component with the greatest magnitude is found in the plane parallel to the surface, and so the light is said to be partially polarized in this plane.

The two diagrams in Figure 9.8 can be combined into one, as shown in Figure 9.9. In this diagram, a dot indicates an electric field into or out of the page, and a double-headed arrow an electric field along the plane of incidence.

The degree to which the reflected ray is polarized depends on the angle of incidence. Consider an unpolarized light ray incident

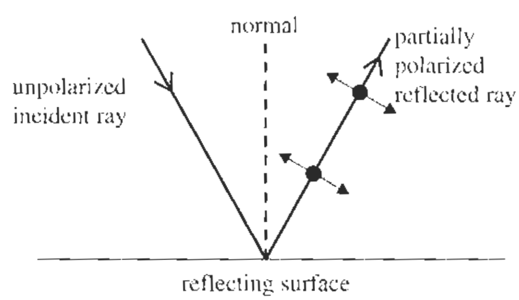

Figure 9.9 A double-headed arrow represents an electric field in the plane of incidence. A dot represents an electric field into or out of the page (i.e. polarizations parallel to the reflecting surface).

on a partly reflecting non-metallic surface (which is transparent to some extent, so that some light is transmitted). There exists a particular angle of incidence, called the polarizing angle or **Brewster angle**, for which the reflected ray is 100% polarized along a plane parallel to the reflecting surface (see Figure 9.10).

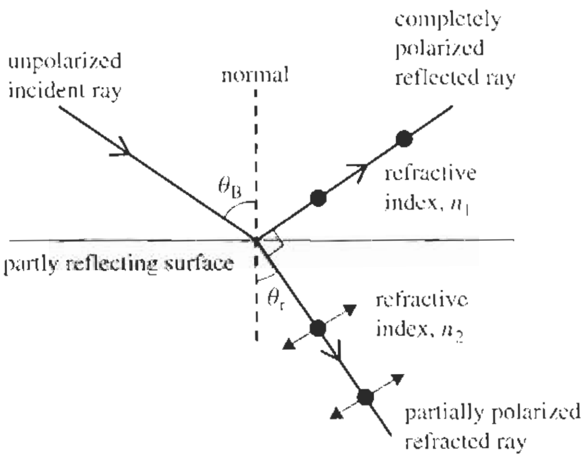

Figure 9.10 When the angle of incidence equals the Brewster angle (polarizing angle), the reflected ray is totally polarized in a plane parallel to the reflecting surface. Notice that the refracted ray is partially polarized.

In 1812, Sir David Brewster (who also invented the kaleidoscope) found experimentally that, when the reflected ray is 100% polarized, *the angle between the reflected ray and the refracted ray is 90°*.

The Brewster angle θ_B is determined by the refractive indices of the two media separated by the partly reflecting surface. Let the refractive index in the medium from which the ray is incident be n_1 and the refractive index of the medium the ray is entering be n_2. Then, the angle of incidence is θ_B and the angle of refraction is $90° - \theta_B$. Applying Snell's law we find:

$$n_1 \sin\theta_B = n_2 \sin(90° - \theta_B)$$
$$= n_2 \cos\theta_B$$
$$\Rightarrow \tan\theta_B = \frac{n_2}{n_1}$$

Brewster's law states that

$$\tan\theta_B = \frac{n_2}{n_1}$$

In particular, if the ray is incident from air ($n_1 = 1$), then $\tan\theta_B = n_2$.

Example question

Q2

Calculate the Brewster angle for light incident on the surface of water. The refractive index of water is 1.33.

Answer

Applying $\tan\theta_B = \frac{n_2}{n_1}$ we find

$$\tan\theta_B = \frac{1.33}{1.00} \Rightarrow \theta_B = \tan^{-1} 1.33 = 53.1°$$

The angle of refraction θ_r for an angle of incidence equal to the Brewster angle θ_B is expected to be $90° - \theta_B = 36.9°$. Indeed, from Snell's law

$$n_1 \sin\theta_B = n_2 \sin\theta_r$$

$$1.00 \times \sin 53.1° = 1.33 \times \sin\theta_r$$

$$\sin\theta_r = 0.601$$

$$\theta_r = 36.9°$$

Optical activity

Consider two polarizers (analysers) whose transmission axes are at right angles to each other, as shown in Figure 9.11. No light is expected to be transmitted through the second polarizer (analyser). However, if we place certain sugar solutions between the two polarizers (analysers), light does get transmitted.

This is because the sugar solution has *rotated* the plane of polarization of the light entering it, so that this light, entering the second polarizer (analyser), has a component of electric field along the second transmission axis.

The rotation of the plane of polarization is called *optical activity* and materials showing this phenomenon are said to be *optically active*.

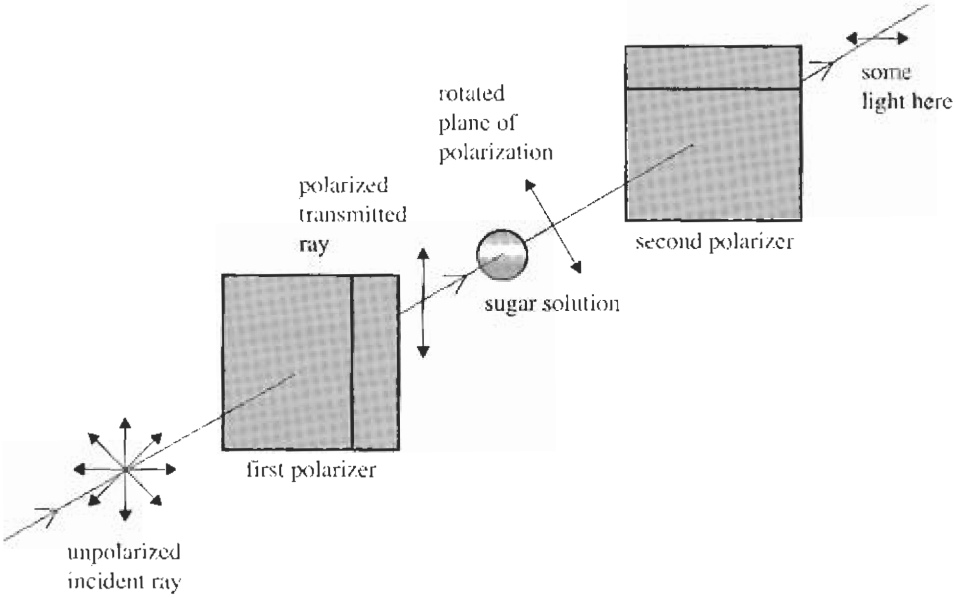

Figure 9.11 No light would normally pass through the two polarizers at right angles to each other. The presence of the sugar solution rotates the plane of polarization, so that light does get through.

The phenomenon of **optical activity** was first studied by the French physicist Dominique Arago in 1811. The phenomenon is exhibited by very many substances, such as organic compounds, notably sugar solutions, tartaric acid and turpentine, as well as many substances in crystal form, such as quartz. The angle by which the plane of polarization rotates depends on the distance travelled within the material and the wavelength of light used. In quartz, the angle rotates by approximately 22° for every millimetre travelled by yellow light. It is an interesting fact that some substances will rotate the plane of polarization clockwise (as we face the source of light) and others in an anticlockwise sense. This has fascinating applications in biology and biochemistry.

In the simple arrangement of Figure 9.11, the angle by which the plane of polarization rotates can easily be measured simply by rotating the second polarizer (analyser) until no light gets transmitted. The angle by which the polarizer (analyser) must be turned is equal to the angle of rotation by the optically active substance.

Practical applications of polarization

Stress analysis

It has been discovered (by Sir David Brewster in 1816) that certain materials that are not normally optically active become so if subjected to stresses. The degree to which the substance becomes optically active is proportional to the stress. A complicated pattern will be seen when a piece of plastic, under stress, is placed in between two polarizers at right angles to each other (Figure 9.12). Examination of the pattern reveals information about how the stress varies in the material. You can sometimes see patterns of coloured light on the windshield of a car if the glass has not been properly installed and is under stress.

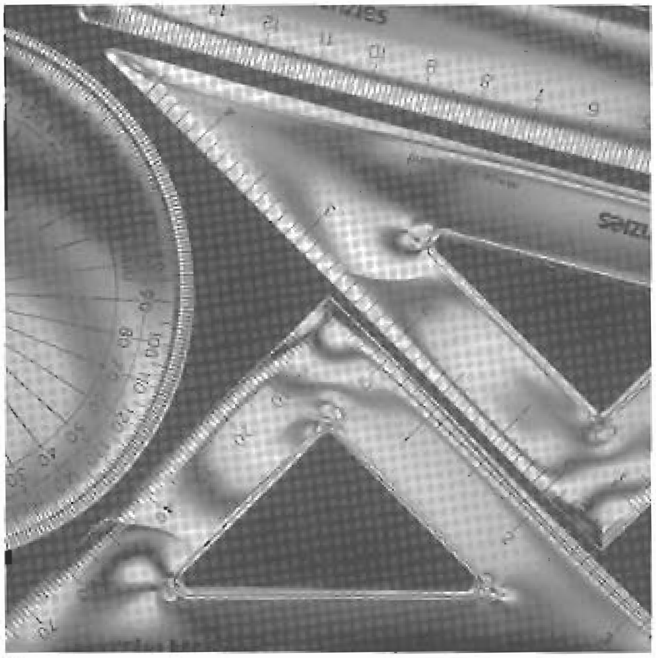

Figure 9.12 Plastic under stress.

Measuring solution concentrations

The amount of rotation of the plane of polarization in a sugar solution depends on the concentration of the solution. An early application of polarization has been to measure concentrations in solutions by measuring the angle of rotation of the polarization plane.

Liquid crystal displays

A more modern application is in liquid crystal displays (LCDs). These can be seen on calculators, watches and the elegant, thin, flat computer and TV screens available today.

An LCD consists of a surface of tiny rectangles called pixels (picture elements). Each pixel has liquid crystals in between two glass plates. The liquid crystals are relatively long, thin molecules that attract each other rather weakly. The first glass plate has very thin (the order of magnitude is 1 nm) slits or scratches along its surface so that the long, rod-like molecules align themselves with the slits. The other glass plate has similar slits but is rotated by 90° with respect to the first. Thus if the molecules next to the first glass plate are, say, vertical, those in contact with the other plate will be horizontal. The molecules in between will therefore, because of the forces between them, slowly change orientation from vertical to horizontal (see Figure 9.13).

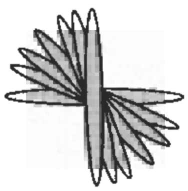

Figure 9.13 The liquid crystal molecules are long and attract each other weakly. Here they form a line that gradually twists as we move into the plane of the page. The orientation of the molecules eventually becomes horizontal at the back plate.

Suppose now that a polarizer with its axis vertical is placed in front of the top glass plate. The transmitted light will be vertically polarized. As the light moves from molecule to molecule, its plane of polarization changes so as to be aligned with the orientation of the molecules. By the time the light reaches the back plate, the plane of polarization has rotated by 90°. If a second polarizer is placed behind the back plate with an axis of transmission at 90° with respect to the first polarizer, the light will simply go through and the pixel will be bright.

However, if a potential difference is established between the two glass plates, the molecules will tend to align their long axes with the electric field. The light reaching the back polarizer will therefore not be able to go through since it will still be vertically polarized. The pixel will then be dark (Figure 9.14).

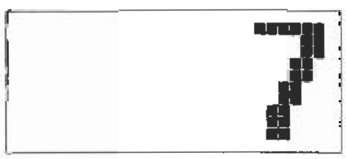

Figure 9.14 The number 7 on a calculator LCD is formed from dark pixels to which a voltage has been applied. The rest of the pixels are bright.

The idea, then, is to apply a voltage to certain pixels so they will appear black against the bright background of those pixels where no voltage is applied. The background can be made to look bright by placing a mirror there to reflect the light that went through the bottom polarizer (Figure 9.15).

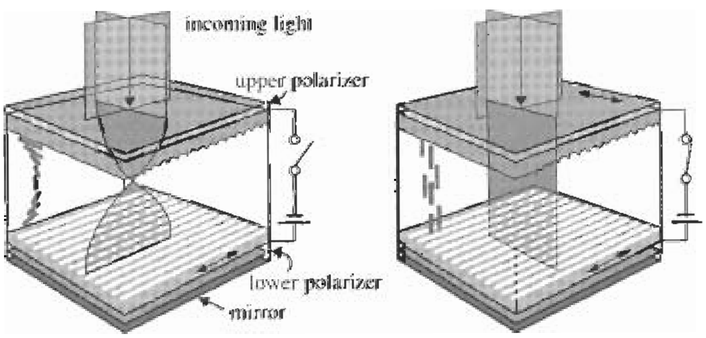

Figure 9.15 In the absence of a voltage between the plates, the light has its plane of polarization rotated, so it can transmit through the lower polarizer. With a voltage, the light is blocked.

Colour can be introduced into LCDs by using green, red and blue filters on sub-pixels. Depending on the relative brightness of the individual sub-pixels, various other colours can be perceived. Computer and TV LCD screens are substantially more sophisticated than the description given above, but the basic principle is the same.

Questions

1 (a) State what is meant by polarized light.
(b) State two methods by which light can be polarized.

2 Explain why only transverse waves can be polarized.

3 Light is incident on an analyser. The transmitted intensity is measured as the orientation of the analyser is changed. In each of the following three outcomes, determine whether the incident light is polarized, partially polarized or completely unpolarized, explaining your answers.
(a) The intensity of the transmitted light is the same no matter what the orientation of the analyser.
(b) The intensity of the transmitted light varies depending on the orientation of the analyser. At a particular orientation, the transmitted intensity is zero.
(c) The transmitted intensity varies as the orientation varies, but it never becomes zero.

4 (a) State Malus's law.
(b) Polarized light is incident on a polarizer whose transmission axis makes an angle of 25° with the direction of the electric field of the incident light. Calculate the fraction of the incident light intensity that gets transmitted through the polarizer.

5 Polarized light is incident on a polarizer whose transmission axis makes an angle θ with the direction of the electric field of the incident light. Sketch a graph to show the variation with angle θ of the transmitted intensity of light.

6 Unpolarized light of intensity I_0 is incident on a polarizer. Calculate, in terms of I_0, the intensity of light transmitted through the polarizer.

7 Unpolarized light of intensity I_0 is incident on a polarizer. The transmitted light is incident on a second polarizer whose transmission axis is at 60° to that of the first. Calculate, in terms of I_0, the intensity of light transmitted through the second polarizer.

8 Unpolarized light of intensity I_0 is incident on a polarizer. A number of other polarizers will be placed in line with the first so that the final transmitted intensity is $\frac{I_0}{100}$. If each polarizer has its transmission axis rotated by 10° with respect to the previous one, how many additional polarizers are required?

9 Light is incident on two analysers whose transmission axes are at right angles to each other. No light gets transmitted. Determine whether it can be deduced if the incident light is polarized or not.

10 Unpolarized light is incident on two polarizers whose transmission axes are parallel to each other. Calculate the angle by which one of them must be rotated so that the transmitted intensity is half of the intensity incident on the *second* polarizer.

11 Unpolarized light is incident on two polarizers. The angle between the transmission axes of the two polarizers is 50°. What fraction of the incident intensity gets transmitted?

12 Two polarizers have their transmission axes at right angles to each other.
(a) Explain why no light will get transmitted through the second polarizer.
(b) A third polarizer is inserted in between the first two. Its transmission axis is at 45° to the other two. Determine whether any light will be transmitted by this arrangement of three polarizers.
(c) If the third polarizer were placed in front of the first rather than in between the two, would your answer to (b) change?

13 (a) State what is meant by the term *Brewster angle* (polarizing angle).
(b) Calculate the Brewster (polarizing) angle for light incident on a liquid of refractive index 1.40.

(c) Calculate the angle of refraction for a ray of light incident on the liquid with an angle of incidence equal to the value you found in (b).

14 Calculate the Brewster (polarizing) angle for light that is

(a) incident on a water–air surface from air;

(b) incident on a water–air surface from water.

Take the refractive index for water to be 1.33.

15 A fisherman is fishing in a lake. Explain why it would be easier for him to see fish in the lake if he was wearing Polaroid sunglasses.

16 Describe the advantage of Polaroid over ordinary sunglasses.

17 You stand next to a lake on a bright morning with one sheet of Polaroid glass. You don't know the orientation of its transmission axis. Suggest how you can determine it. (You may not use other Polaroid sheets with known transmission axes.)

18 State what is meant by

(a) optical activity;

(b) an optically active substance.

19 State two factors that affect the angle of rotation of the plane of polarization by an optically active substance.

20 Plan an experiment that will allow you to measure the concentration of a sugar solution. What do you need to have? What measurements must you make? How will the concentration of an unknown sugar solution be deduced?

21 State practical applications of polarization.

22 Outline the operation of liquid crystal displays.